$R_i = 159^{+42}_{-37}$

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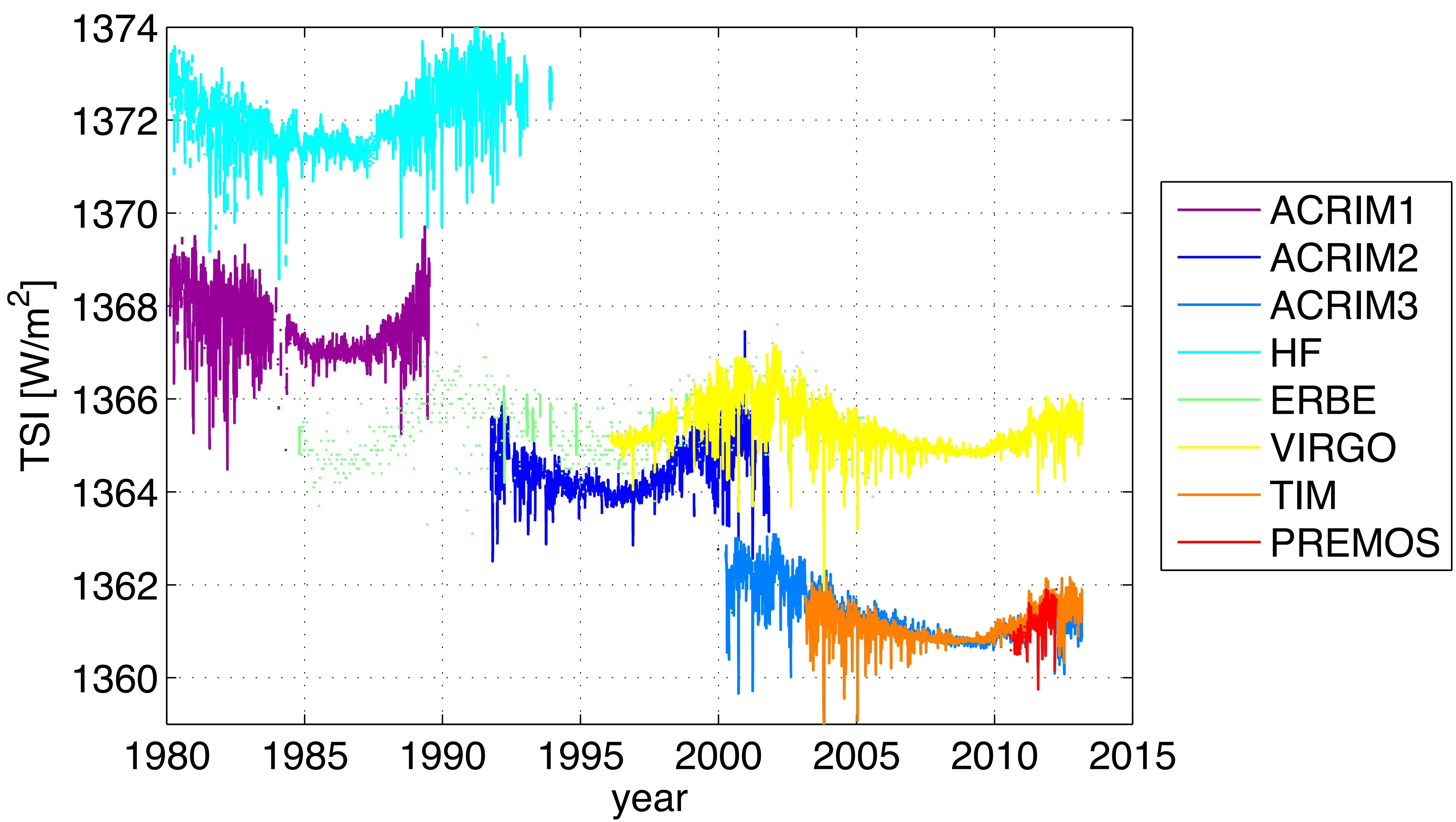
With thanks to Micha Schöll and the SOLID team

Outline

- Realistic confidence intervals are hard to get
- What do we mean by *"confidence interval"* ?
- How can we estimate them ?
  - short-term variations: ok
  - long-term variations: some ideas
- What do they tell us about the underlying physics ?

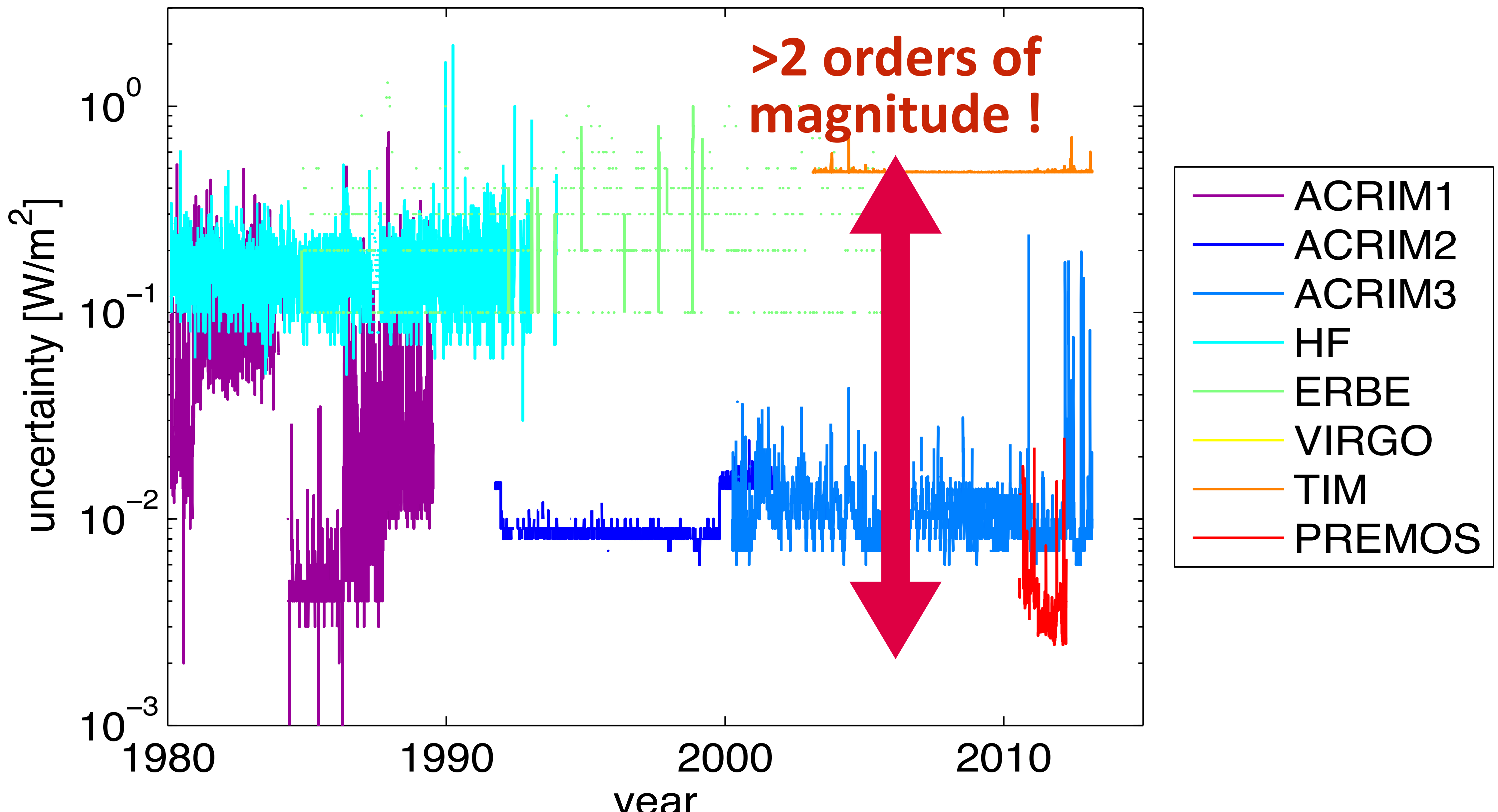
Example : Total Solar Irradiance

- TSI measurements agree on variability, but not on absolute value



Example : Total Solar Irradiance

- Scientists disagree on the level of uncertainty !



Different uncertainties

- Here

confidence interval = degree of belief ($\neq$ error)

- Different contributions
  - random fluctuations in the emergence of sunspots (Poisson)
  - errors in counting the number of sunspots (~Gamma)
  - averaging over various observers (~Gaussian)
  - discretisation error (uniform)
  - systematic errors
  - etc.

Different uncertainties

- We may expect the uncertainties to be some mix

residual error

$$SSN_{obs} = SSN_{true} + \alpha\mathcal{P}(SSN_{true}) + \mathcal{N}(\mu, \sigma^2)$$

Observed SSN — True SSN — Poisson — Gauss

- What do they tell us about the data ?

How do we estimate these $!@##! uncertainties ?

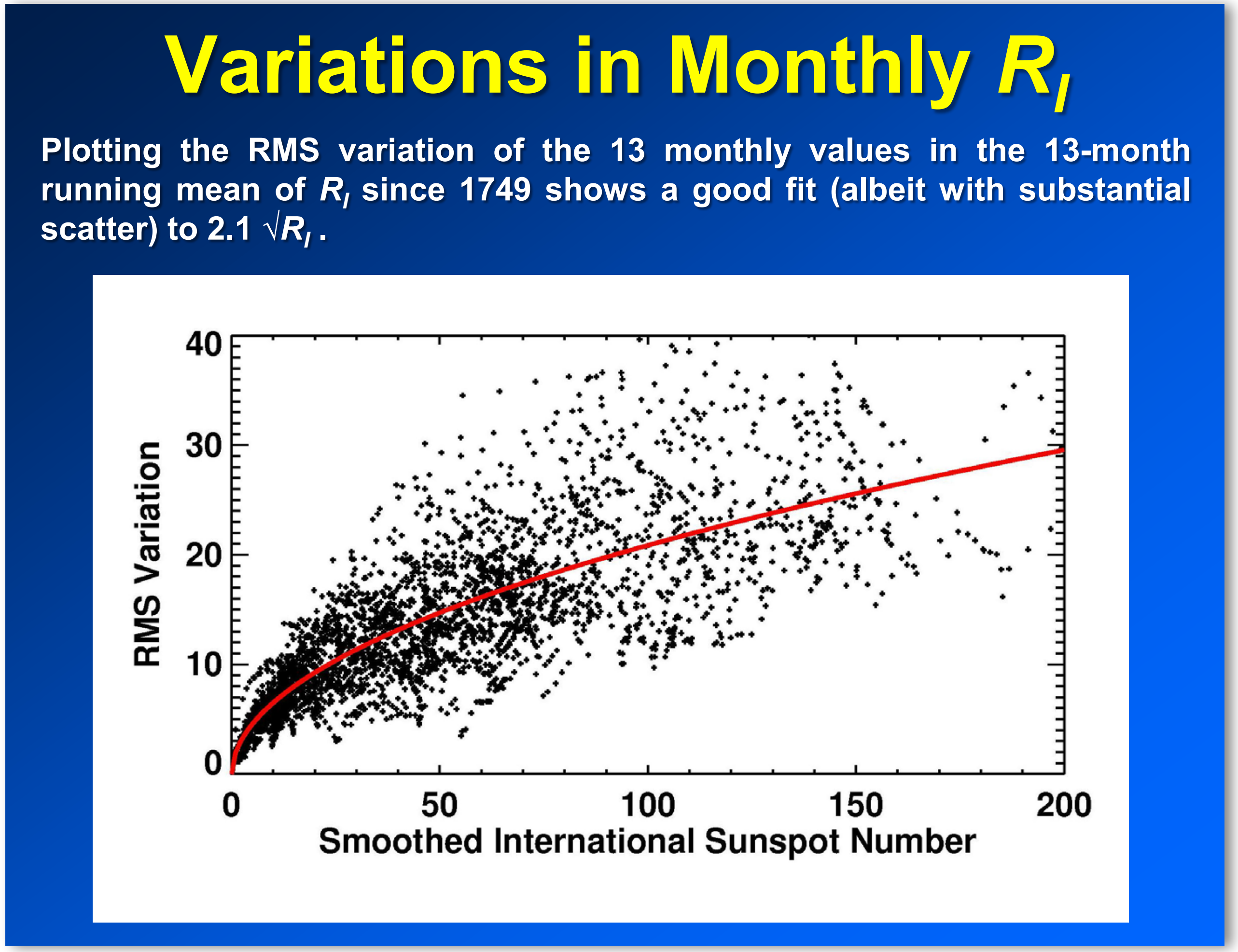
Estimating uncertainties

- Several approaches for determining uncertainties

1. Take daily differences
2. Use power spectral density
3. Use another proxy
4. Model the dynamics of the SSN
5. …

Estimating uncertainties (I)

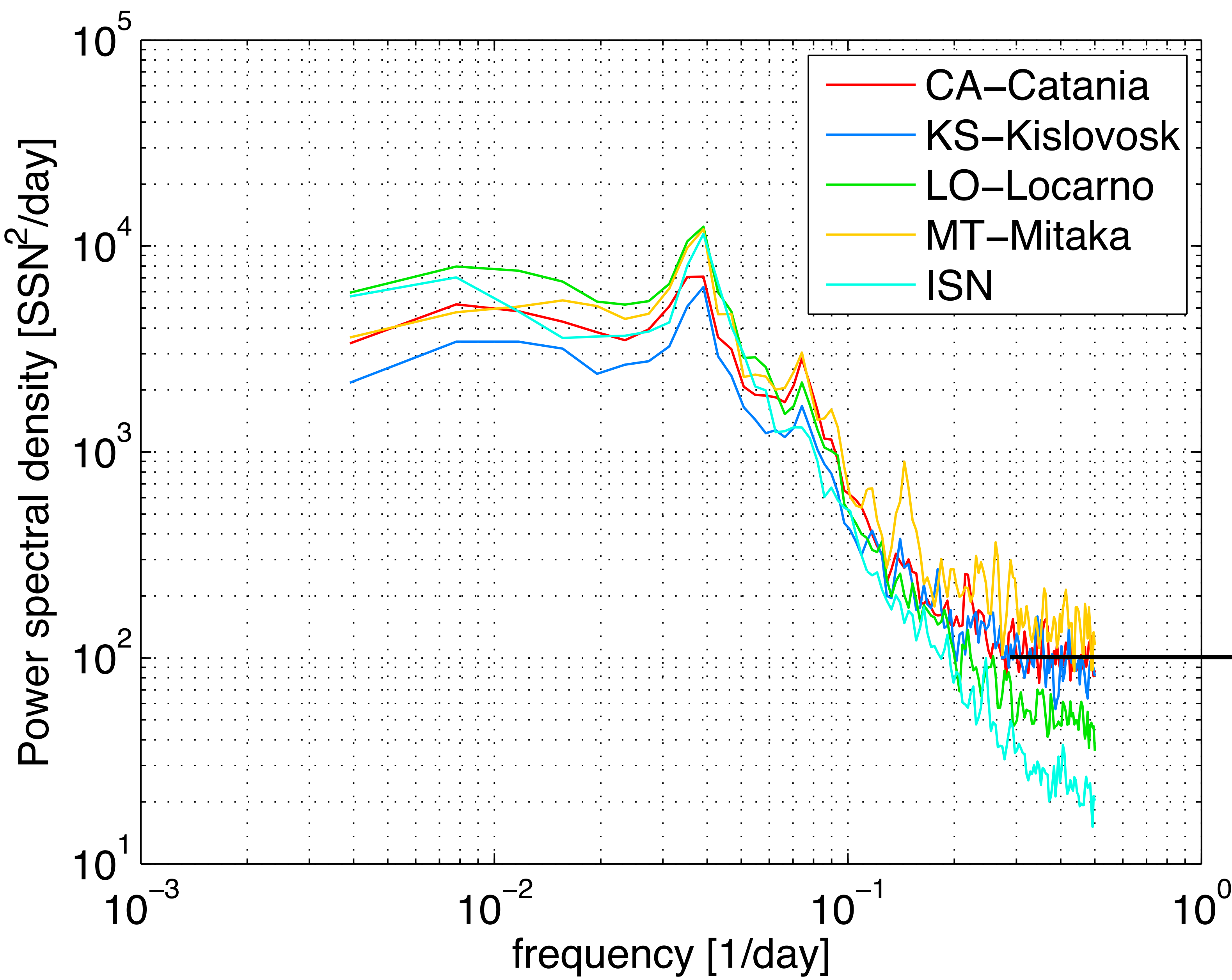
- Assume fluctuations = white noise, and that the SSN is band-limited
  → consider **day-to-day differences** as "noise"



see talk by David Hathaway

Estimating uncertainties (2)

- Assume fluctuations = white noise, and that the SSN is band-limited
  → look for **noise floor in power spectral density**



"noise" floor of Kislovosk = 10 counts

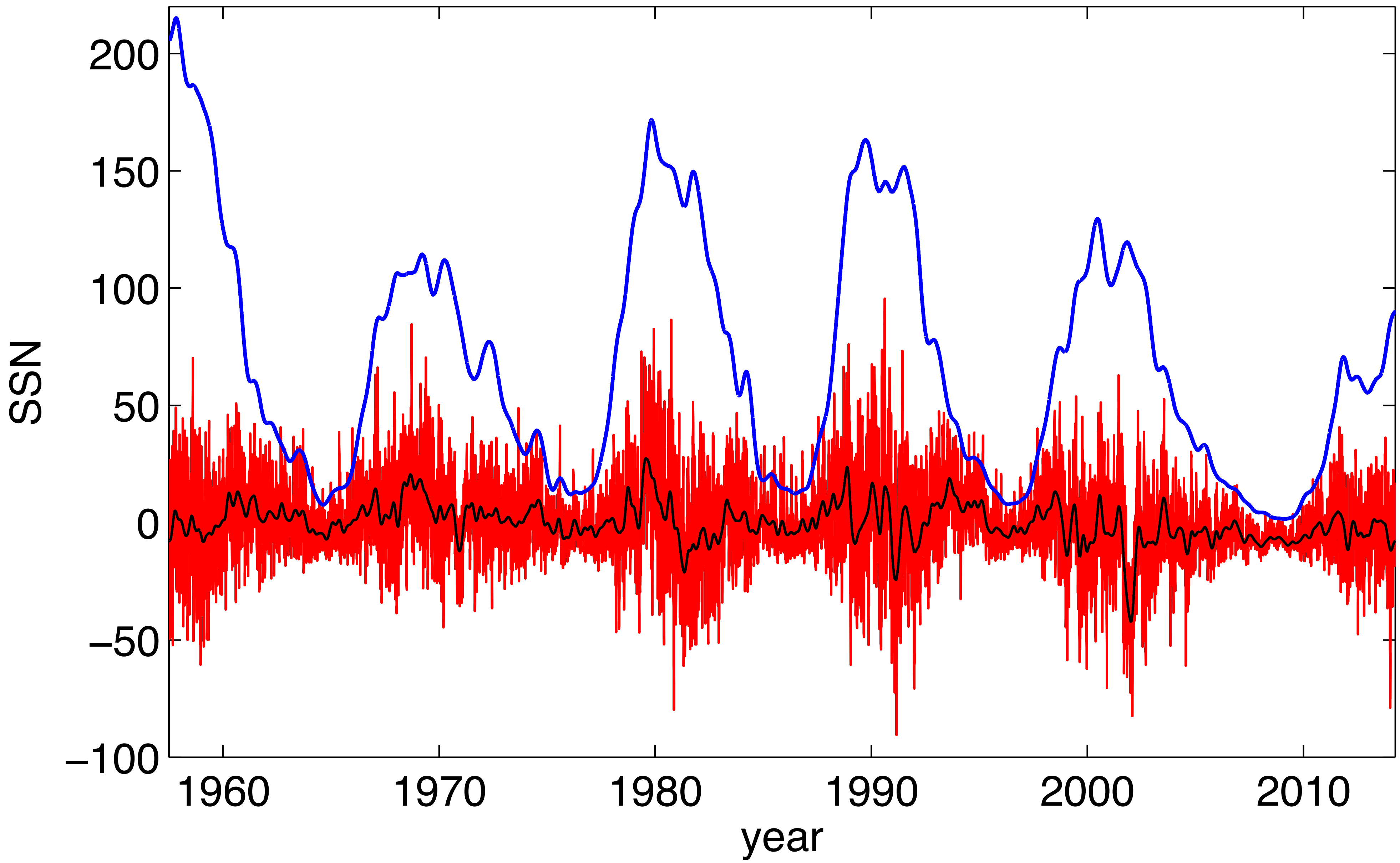
Estimating uncertainties (3)

- Use other proxies to reconstruct the SSN and look at

residual error = SSN - proxy fit

Estimating uncertainties (3)

- **Example** : multiscale reconstruction of the SSN with a linear combination of four radio fluxes (8, 10.7, 15, 30 cm)



SSN (6-month average)

residual error

residual error (6-month average)

Estimating uncertainties (4)

- We use a more pragmatic definition

> Residual error = amount by which today's SSN departs from the value predicted by dynamical system model of the SSN (using past observations)

Estimating uncertainties (4)

- We describe the dynamics of the SSN by using a linear **autoregressive** (AR) model

$$SSN[k+1] = a_0 SSN[k] + a_1 SSN[k-1] + \cdots + a_p SSN[k-p] + \epsilon[k+1]$$

residual error

tomorrow's value

today's value

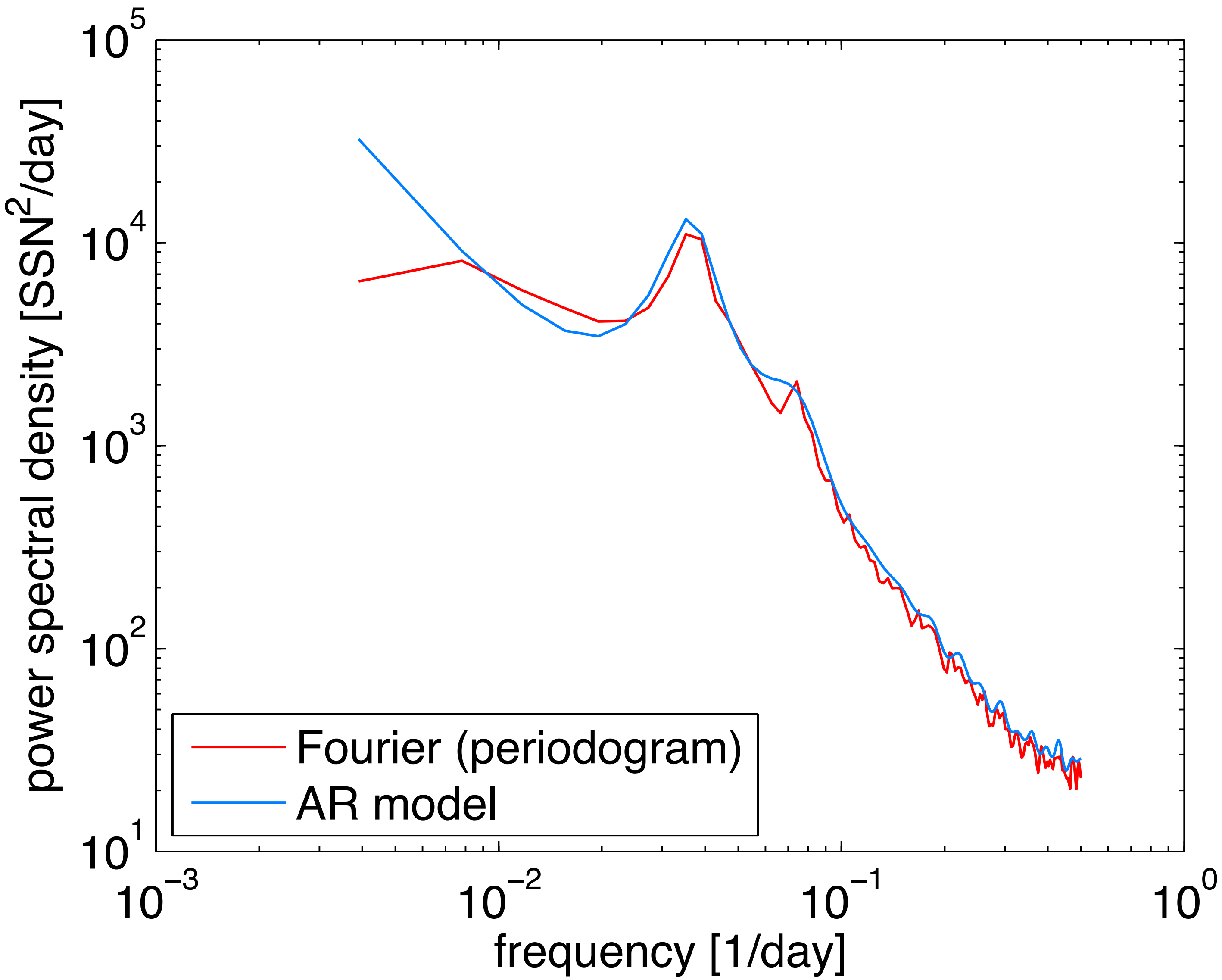
- Various criteria indicate that the optimal model order is p = 6 - 16
- Beware
  - this model has assumptions: linearity, stationarity, ... which are not verified
  - there are better models around: NARMA, etc.

AR model

- Typically, we find for an 6th order model

$$
\begin{aligned}
SSN[k+1] &= 0.9370\ SSN[k] \\
&+ 0.0553\ SSN[k-1] \\
&- 0.0140\ SSN[k-2] \\
&- 0.0019\ SSN[k-3] \\
&- 0.0183\ SSN[k-4] \\
&- 0.0150\ SSN[k-5] \\
&+ 0.0510\ SSN[k-6] \\
&+ \epsilon[k+1]
\end{aligned}
$$

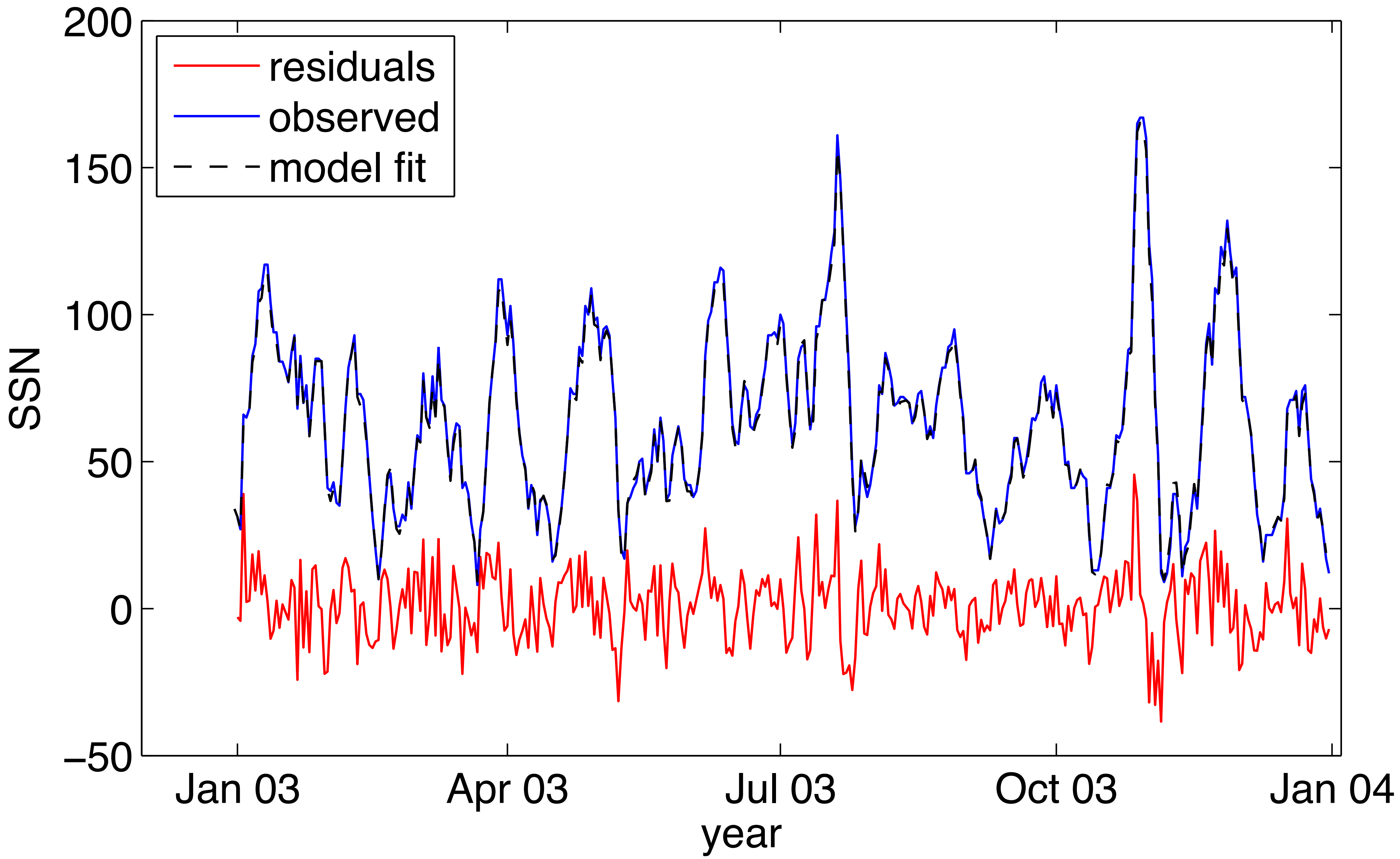
- From these coefficients, we can estimate the power spectral density



The AR model properly describes the dynamics on time scales < few months

Residuals from the AR model

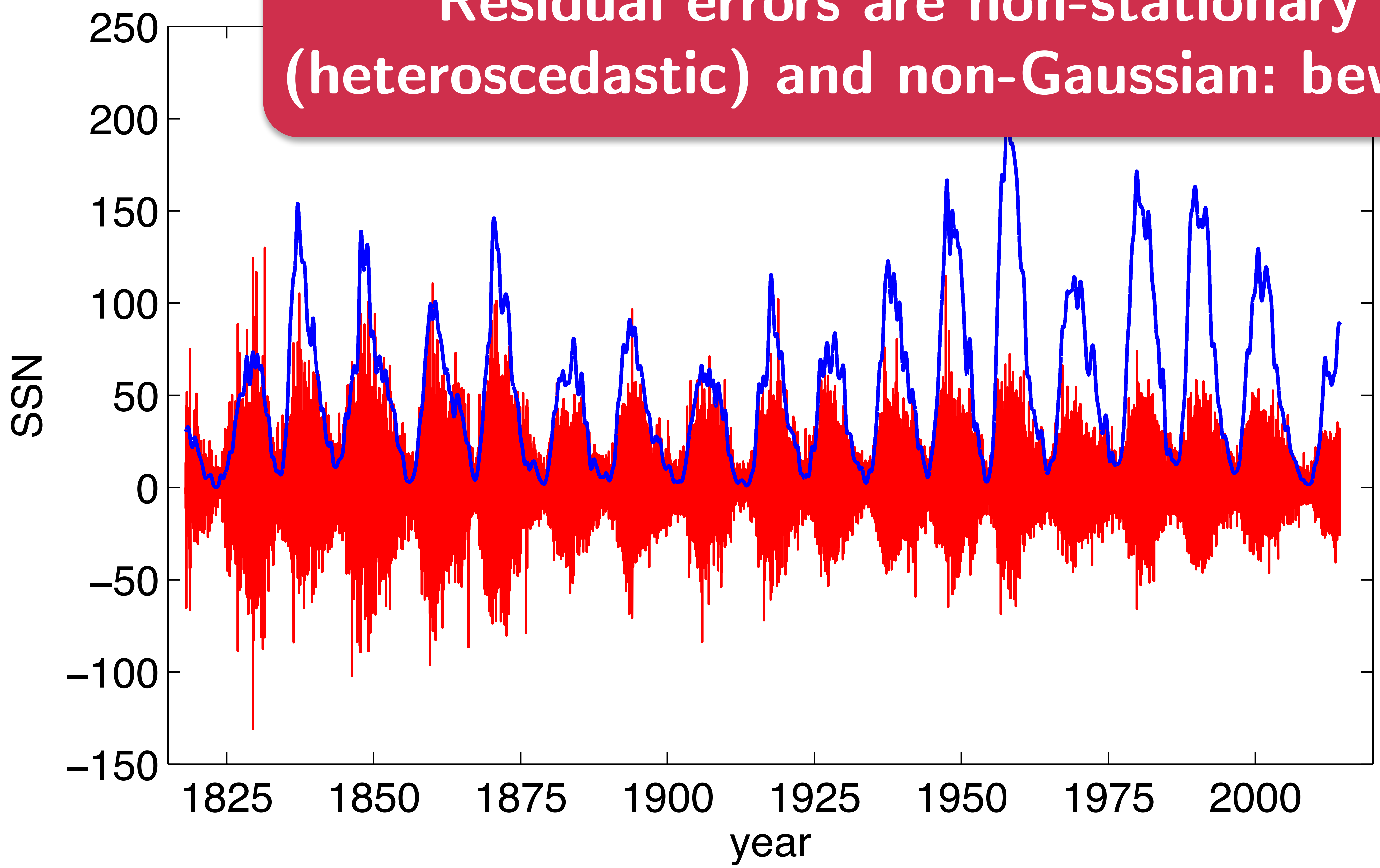
- Residual error from 16th order AR model, applied to ISN (excerpt)



Residuals from the AR model

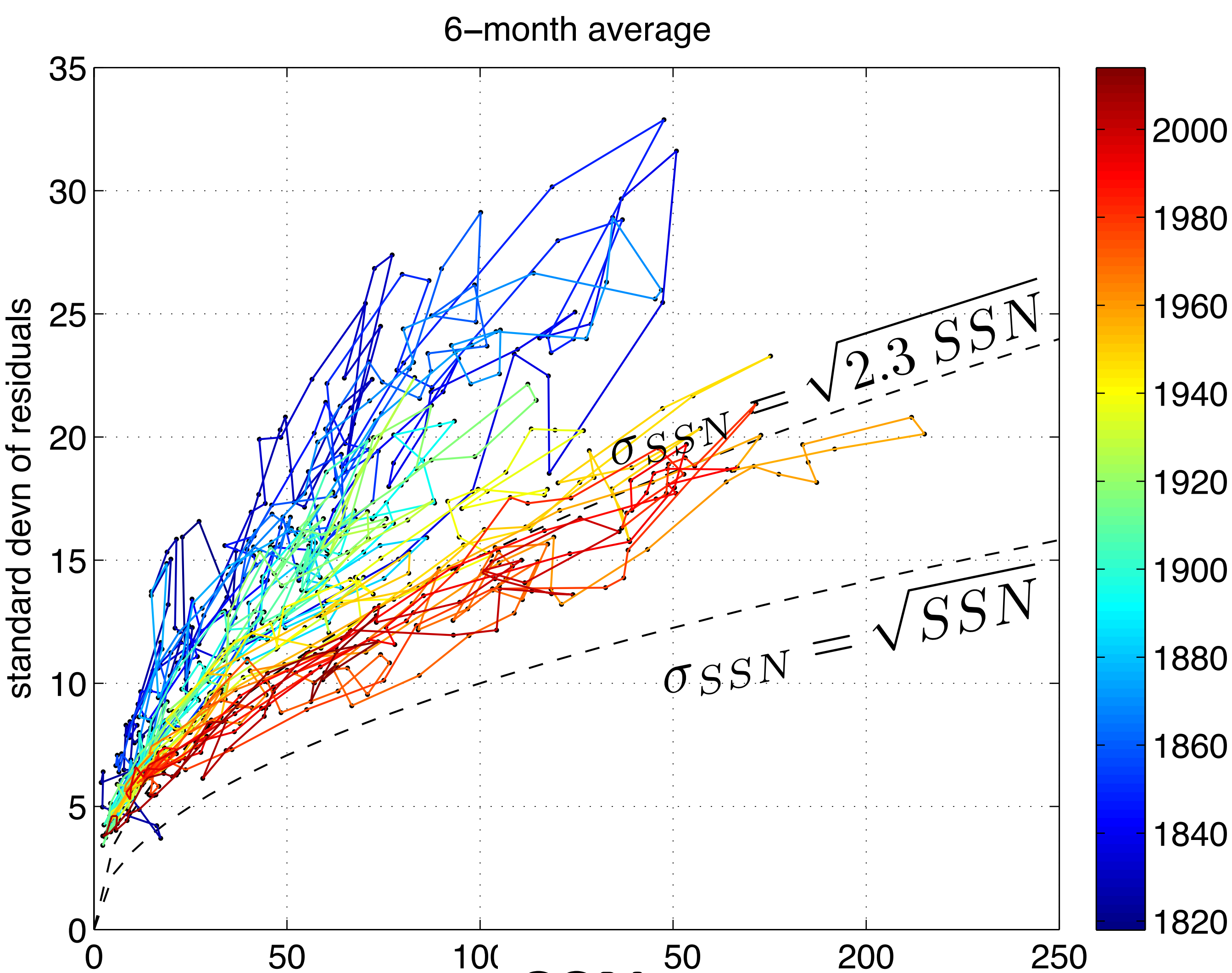
- Residual error from 16th order model, applied to ISN

Residual errors are non-stationary (heteroscedastic) and non-Gaussian: beware!

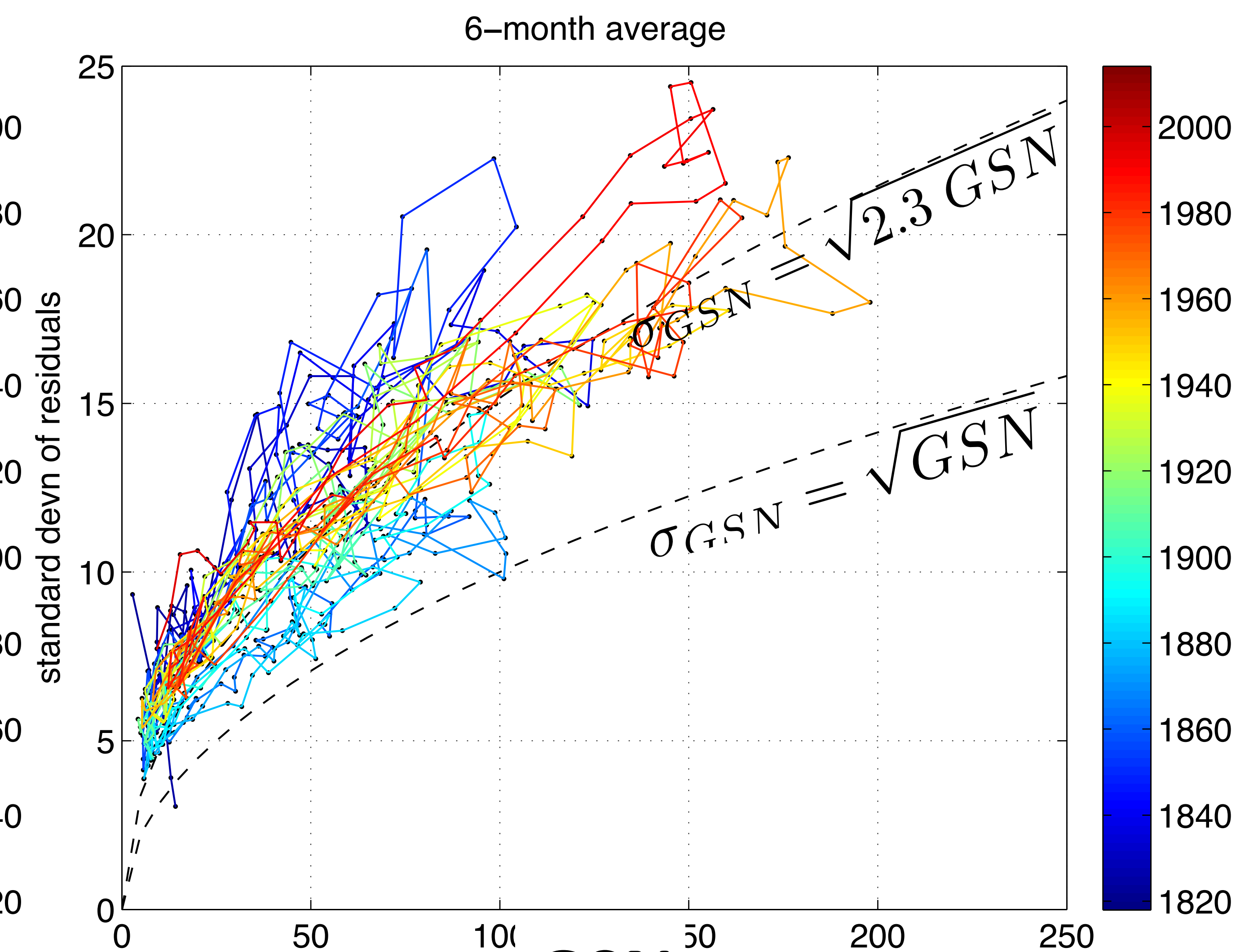


Residuals from the AR model

- Standard deviation of residual is
  - cycle-dependent : smaller for recent cycles
  - scales approximately as $\sigma_\epsilon \propto \sqrt{SSN}$ → Poisson-like
  - same for group sunspot number



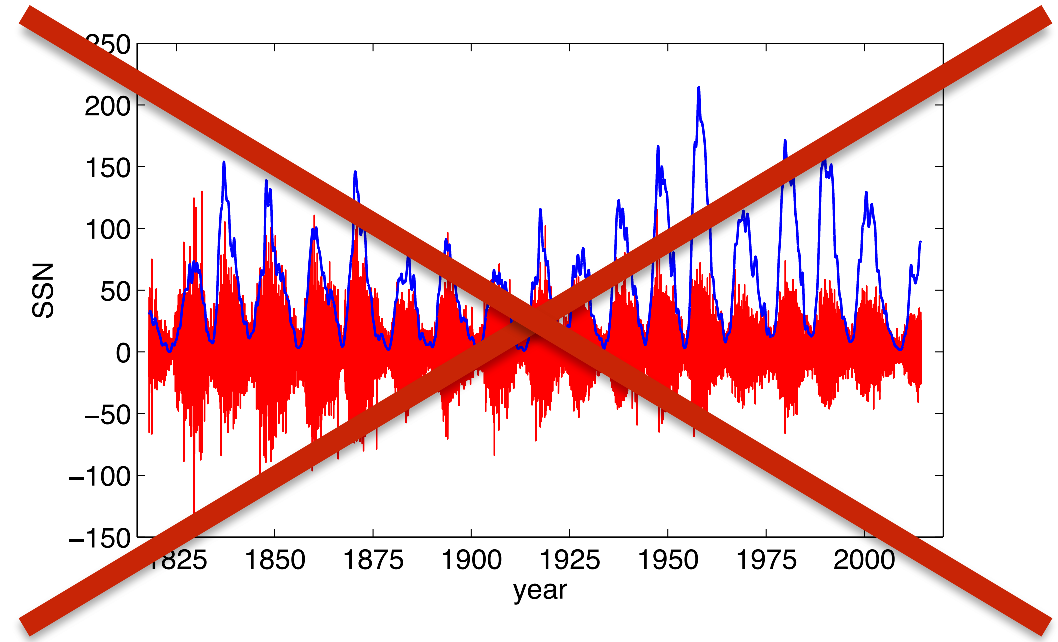
SSN



GSN

Variance stabiliation

- It is essential to **stabilize the variance** in order to be able to proceed → make residual errors stationary in time



- Apply the **Anscombe transform** : If SSN is a mix of Poisson + Gaussian random variables

$$y = \alpha \mathcal{P}(SSN) + \mathcal{N}(\mu, \sigma^2)$$

then

$$y^* = \frac{2}{\alpha}\sqrt{\alpha y + \frac{3}{8}\alpha^2 + \sigma^2}$$

behaves like a Gaussian variable with $y^* \sim \mathcal{N}(\mu', \sigma^2 = 1)$

Variance stabilisation

- Interpretation of the Anscombe transform : if we replace the SSN by

$$SSN^* = 2\sqrt{\frac{SSN}{2.3} + \frac{3}{8}}$$

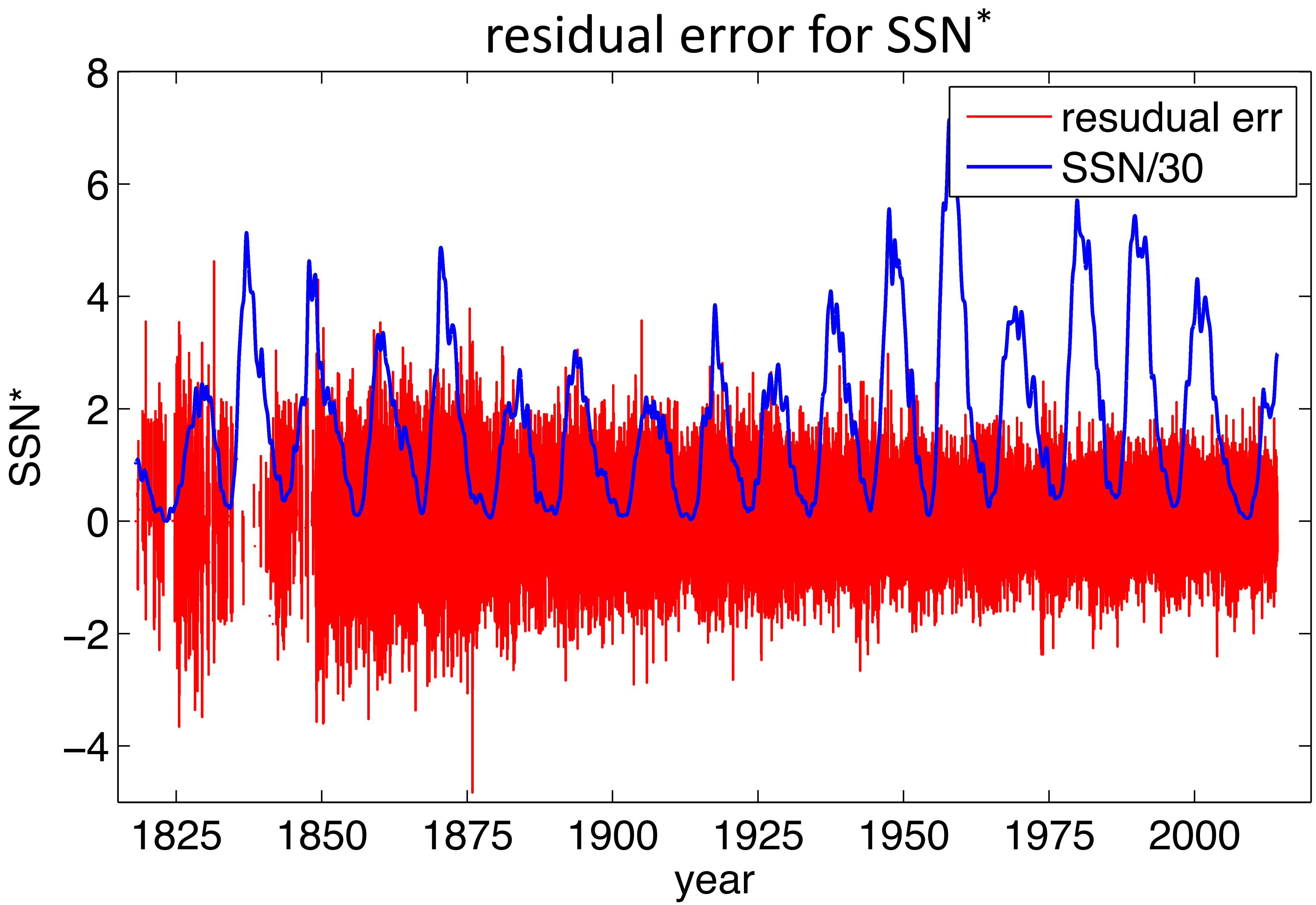
Then the new sunspot number will have a constant and unit variance → SSN* is now stationary and Gaussian!

Thanks to the Anscombe transform, all classical analysis tools can again be used

The Anscombe transform tells us that SSN/2.3 (and not SSN) behaves like a Poisson process

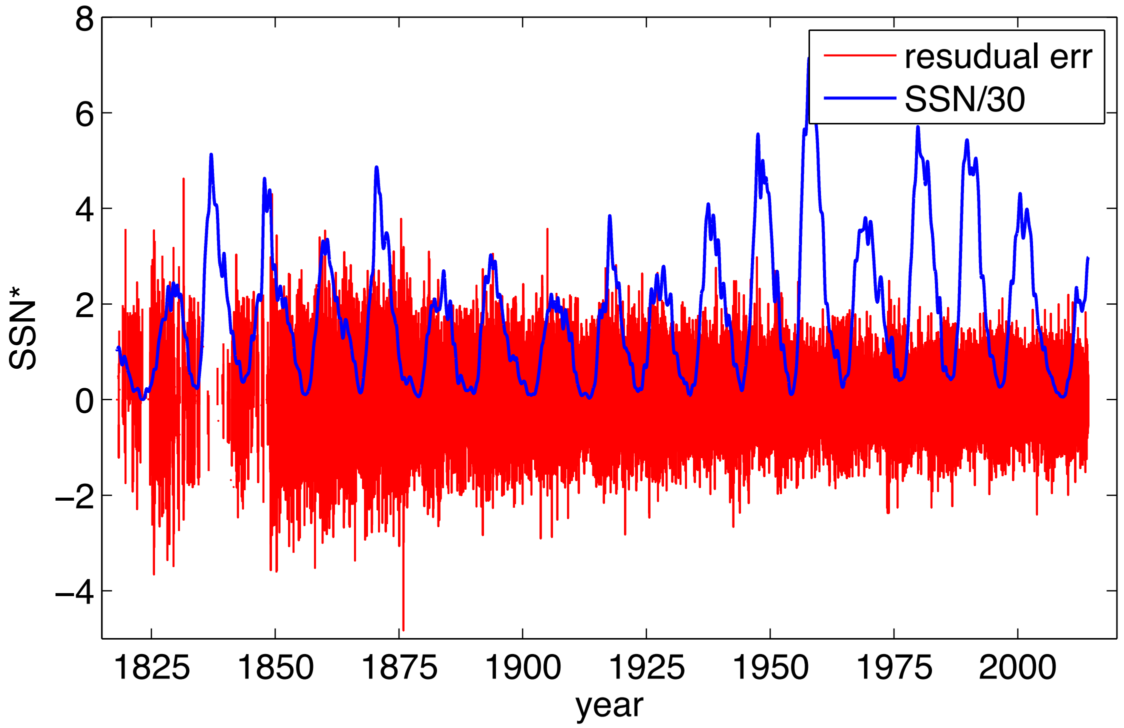
Variance stabilisation

- The variance has now been stabilized



almost cycle independent

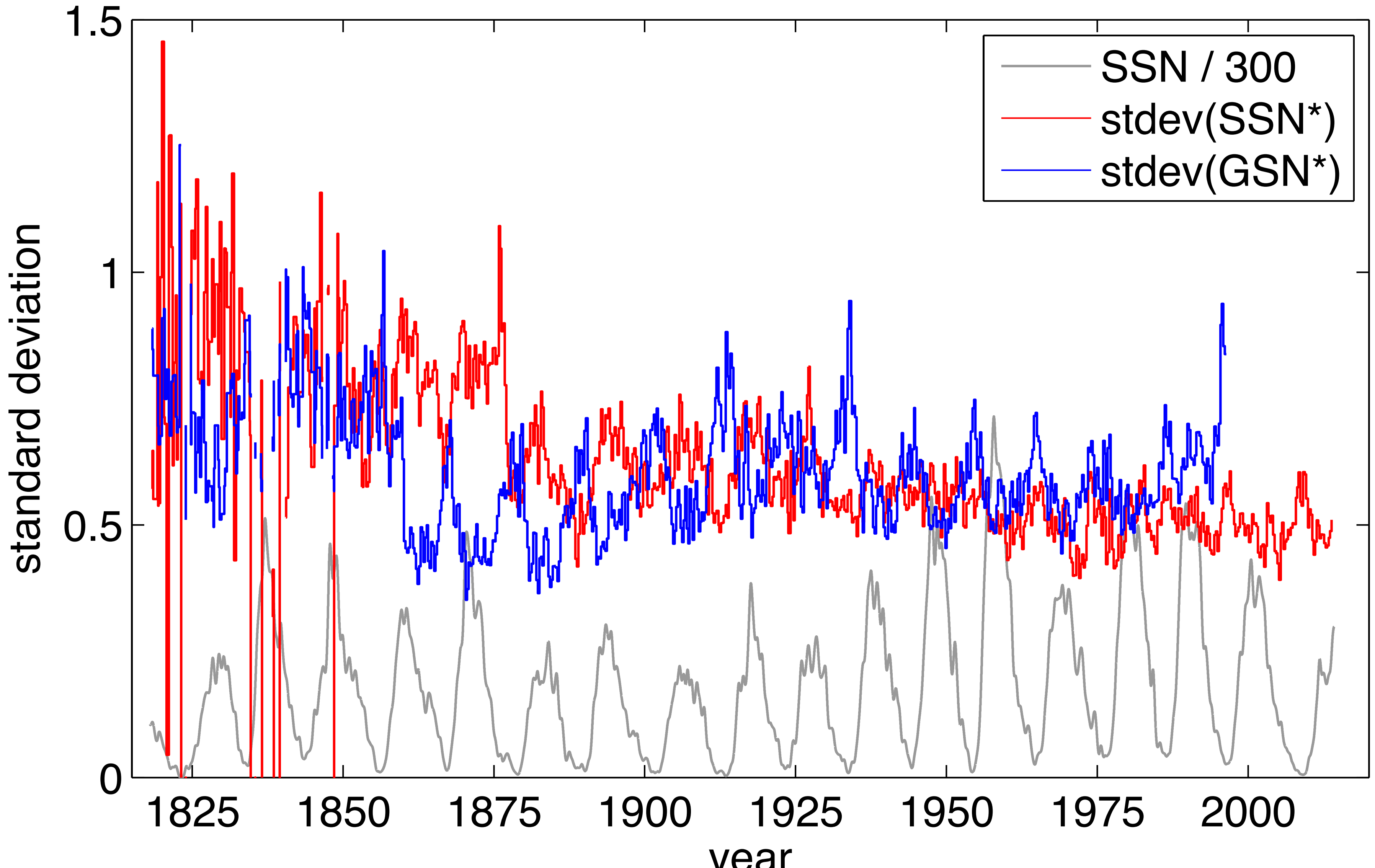
Variance stabilisation



- Key questions
  - How do the GSN and SSN compare ?
  - What does the relative contribution of Poisson/Gaussian fluctuations tell us ?
  - Can we estimate them ?

GSN versus SSN

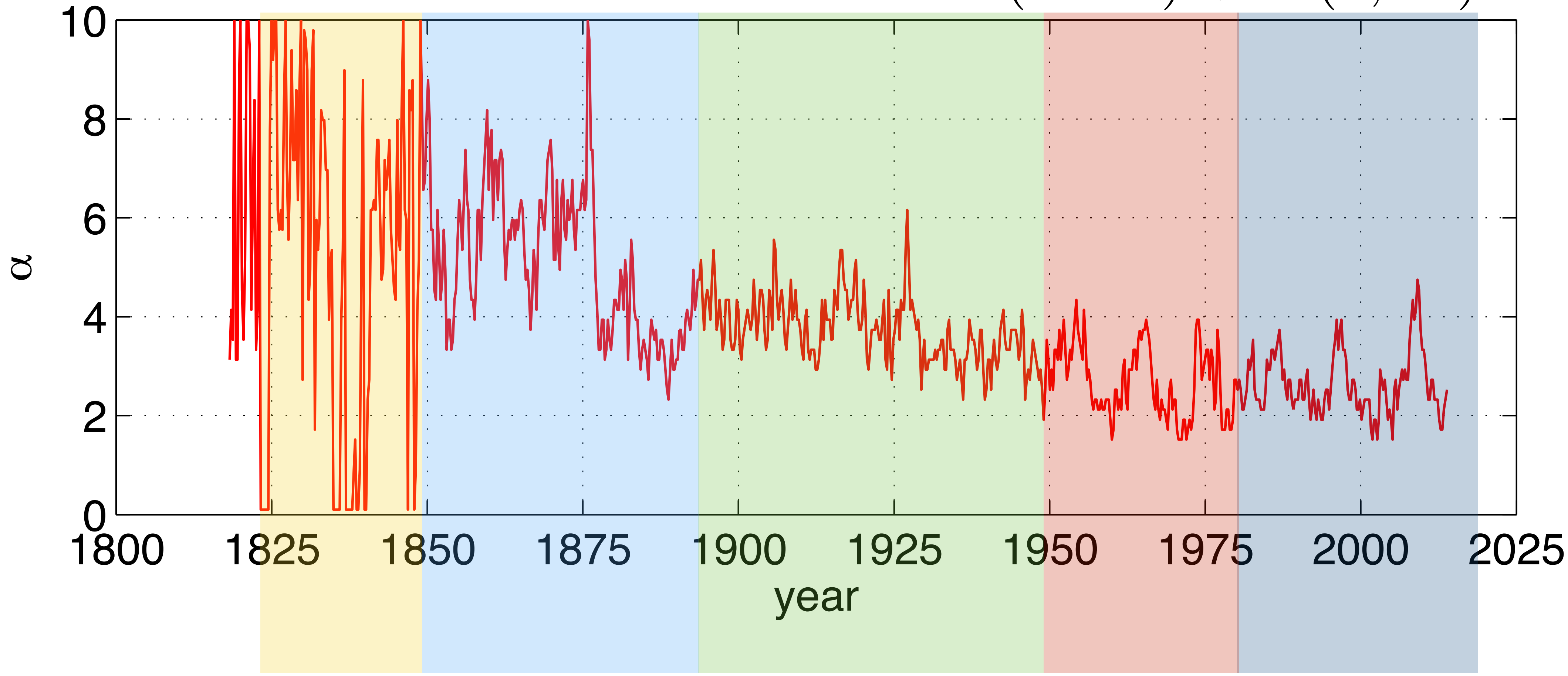
- The errors on the GSN and SSN evolve in different ways
  - the error on the GSN is not as small as expected (averaging effect ?)
  - data collection effects are important



What are the best parameters ?

- Rough estimate of the amplification factor α and the additional error σ

$$SSN^* = \alpha\mathcal{P}(SSN) + \mathcal{N}(0,\sigma^2)$$



Schwabe	Wolf	Wolfer/Br	Waldm.	SIDC
α = 7	α = 5.5	α = 3.5	α = 2.3	α = 2.3
σ = 7	σ = 6	σ = 6	σ = 5	σ = 5

Intermediate conclusion

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- The uncertainties on the SSN are **not stationary in time**
  - most linear regressions with the SSN are flawed because they give too much weight to large values
  - use the Anscombe transform to stabilize the variance

Intermediate conclusion

- The uncertainties on the SSN are **not stationary in time**
  - most linear regressions with the SSN are flawed because they give too much weight to large values
  - use the Anscombe transform to stabilize the variance

- The Anscombe has several advantages
  - the SSN behaves like a mix of Poisson and Gaussian processes

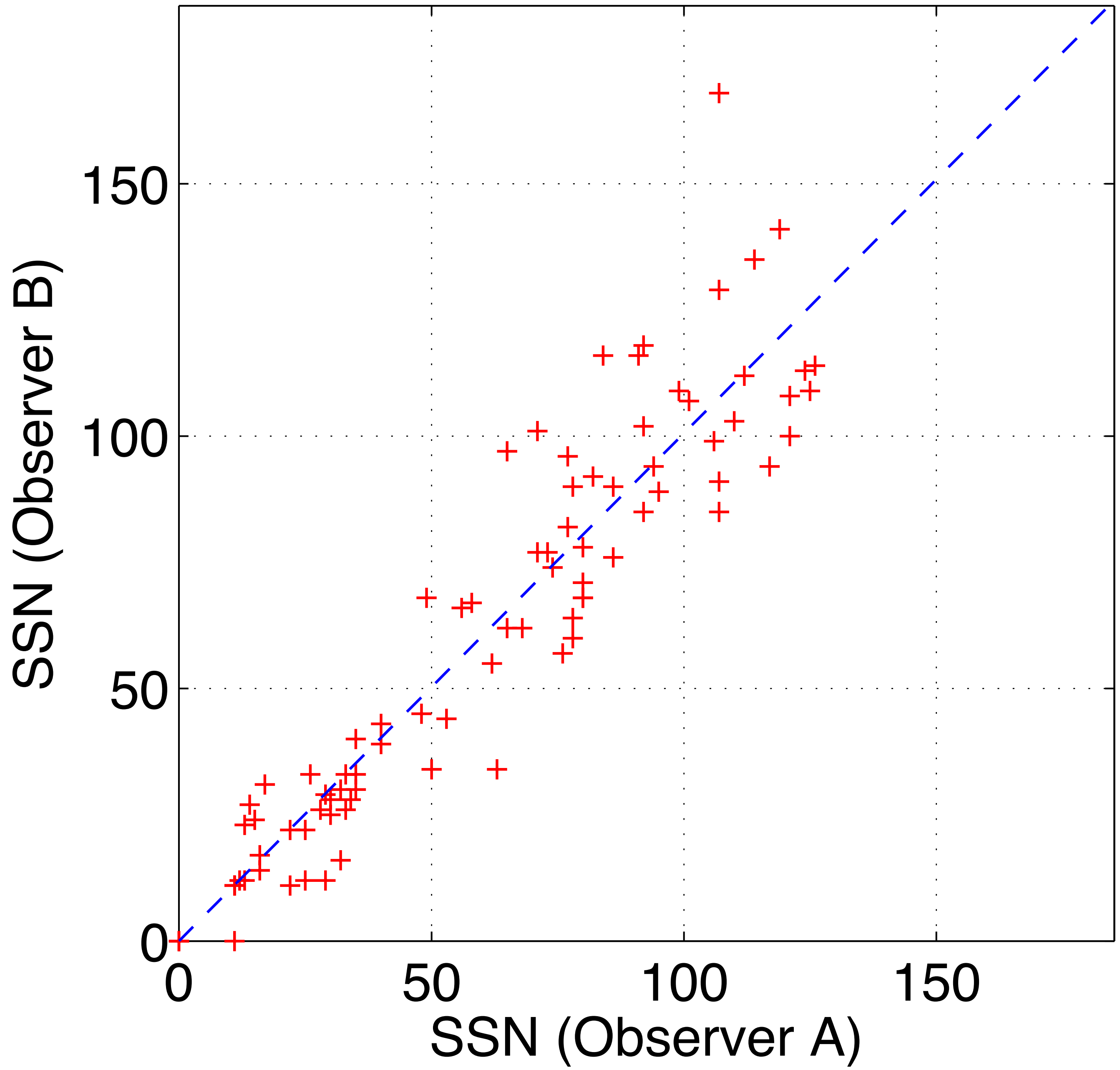
$$SSN \approx 2.3\,\mathcal{P}(SSN_{true}) + \mathcal{N}(0, \sigma^2 \approx 25)$$

  - we now have a sound estimate for the confidence intervals
  - these coefficients change over time

What flaws ?

Flaws in linear regression: example

- What is the ratio between two observers ?

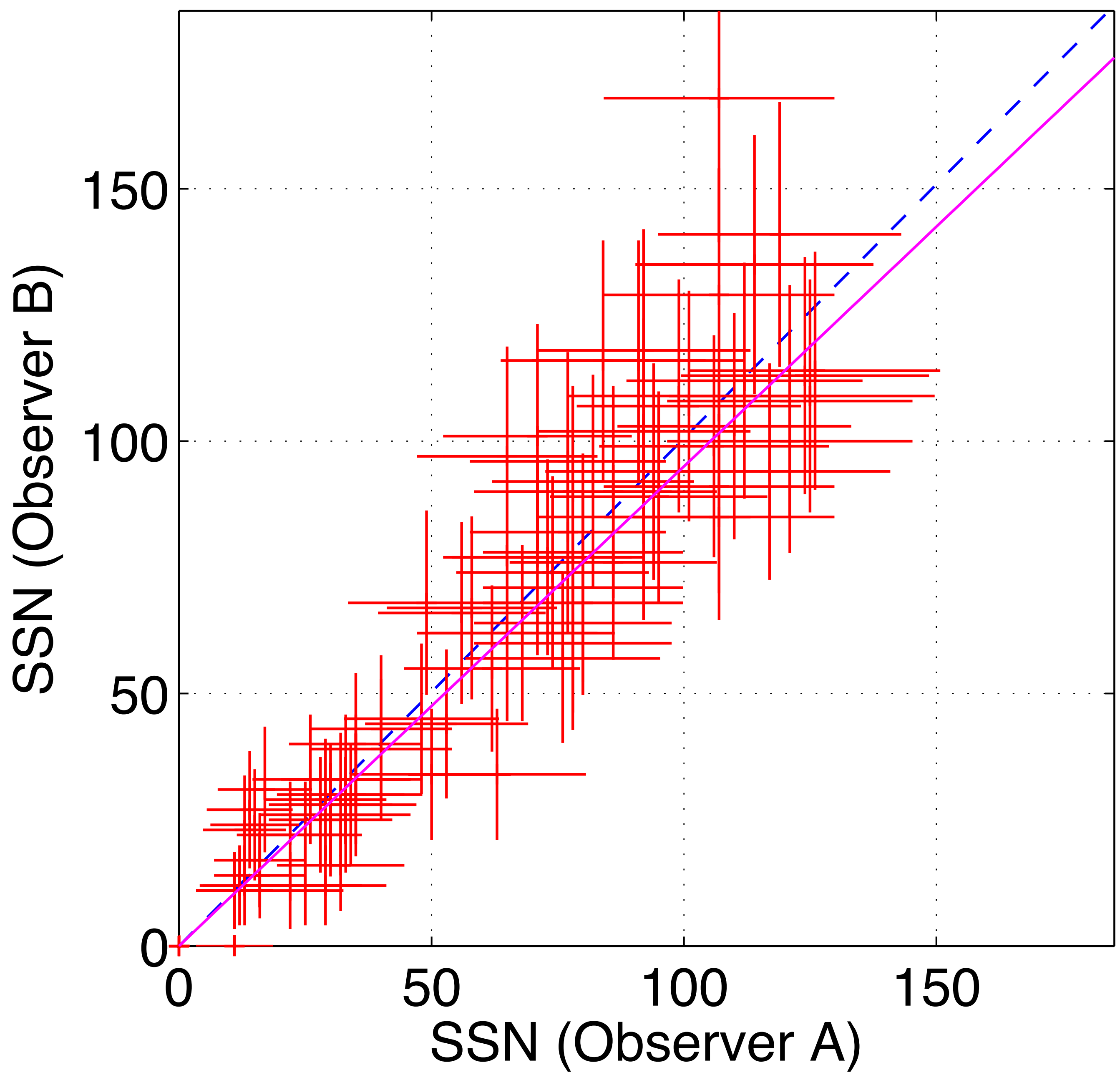


c = 1.006 ± 0.022

obtained by simple least-squares fit, ignoring errors on A and B

Flaws in linear regression: example

- The same, with error confidence intervals for both



Old value

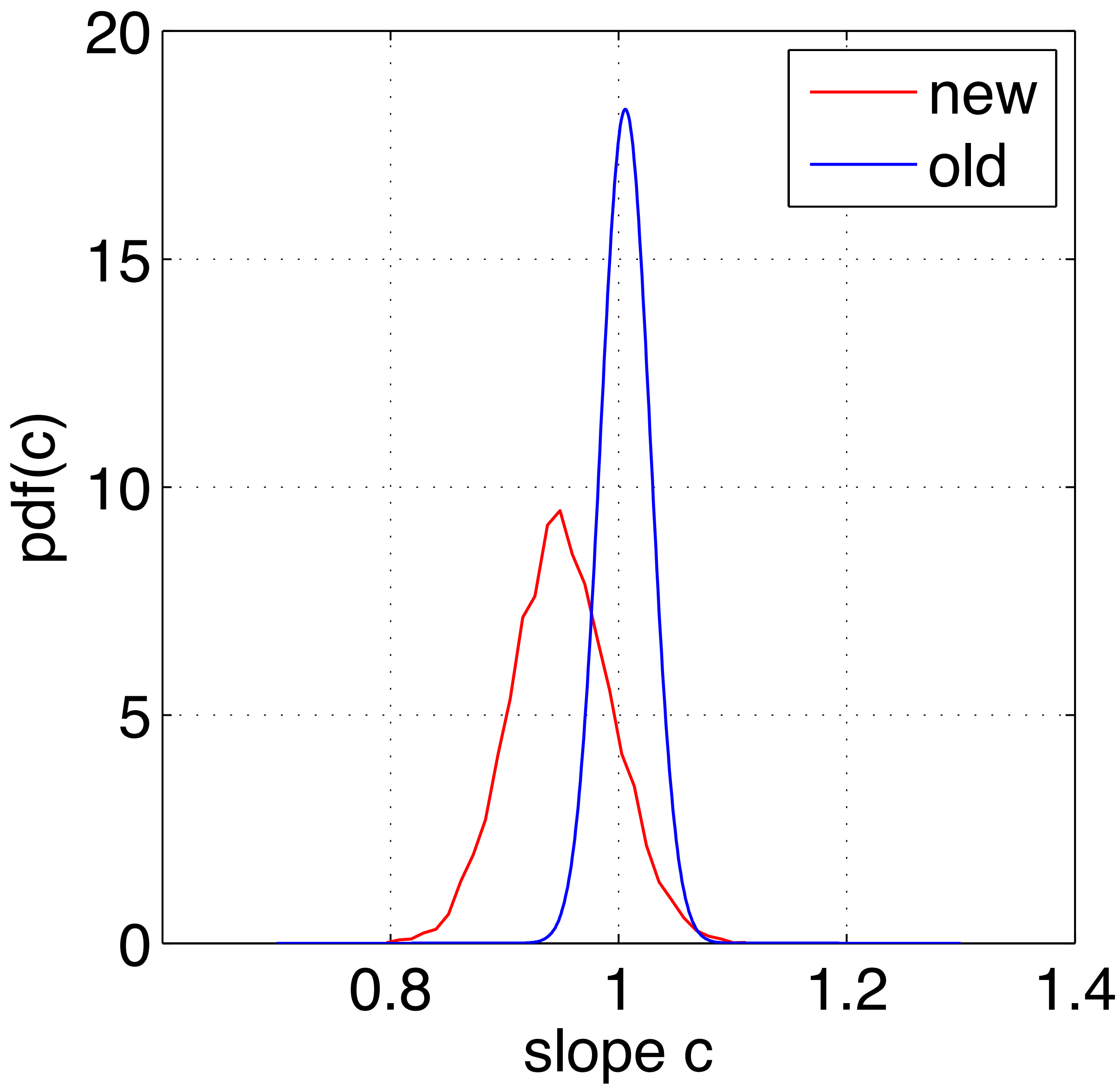
c = 1.006 ± 0.022

New value

c = 0.950 ± 0.044

Flaws in linear regression: example

- Probability distributions of the slope *c* differ because the second model includes uncertainties on the observations



Great care is needed when making linear regressions with noisy data !

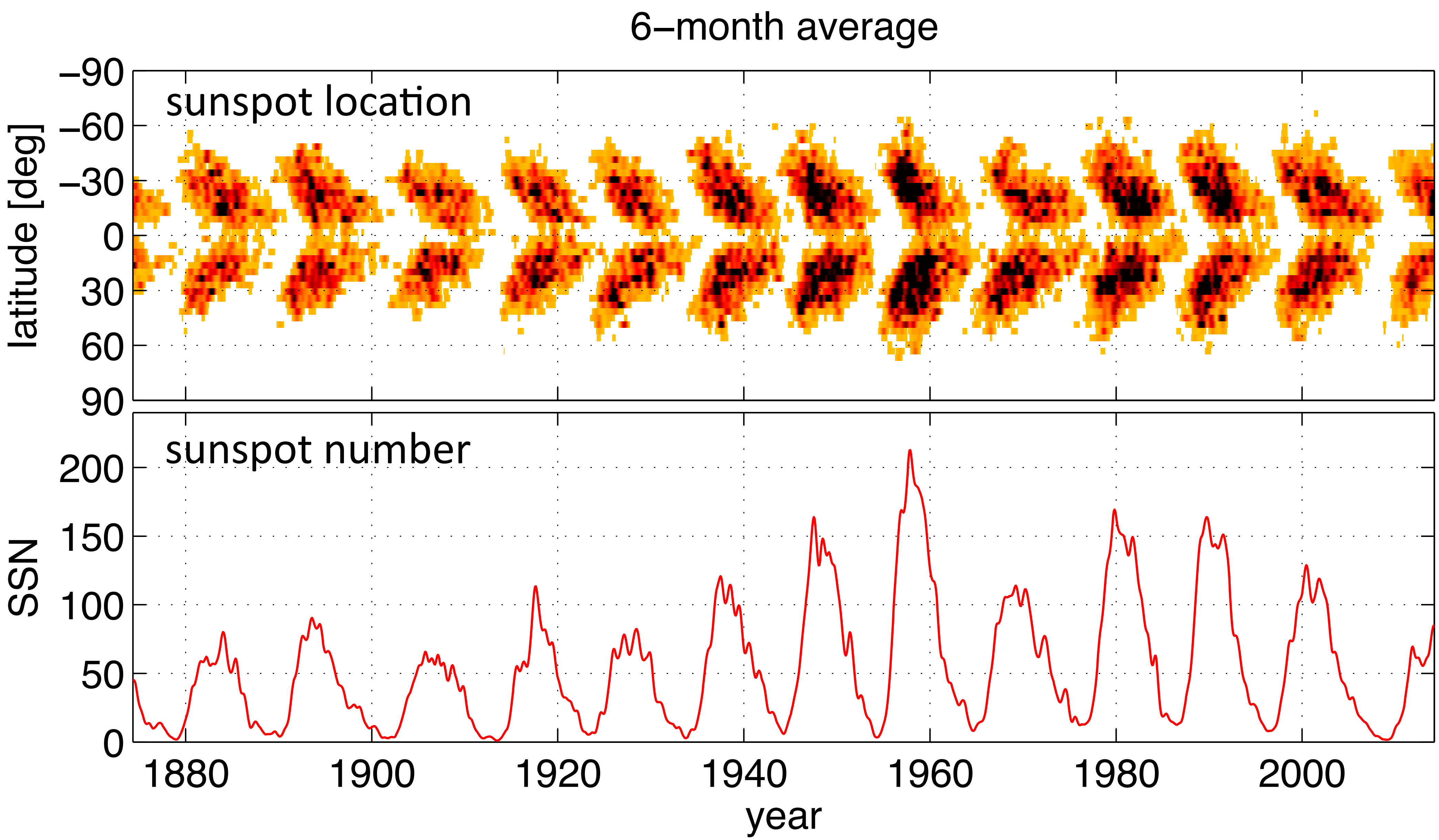
What about long time scales ?

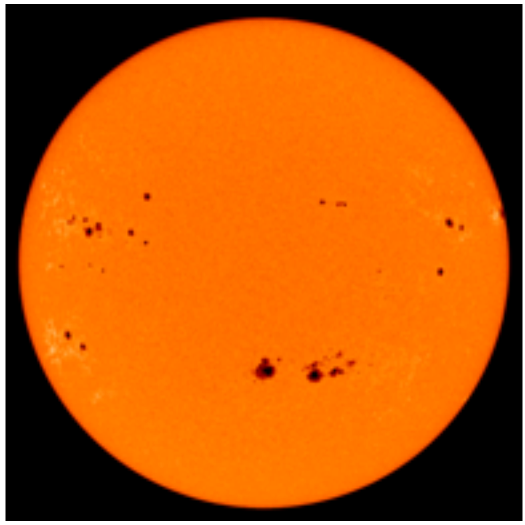
Long time scales

- Uncertainties for long time scales are challenging !
- But there are some sanity checks
  - use the Butterfly diagram

Butterfly diagram

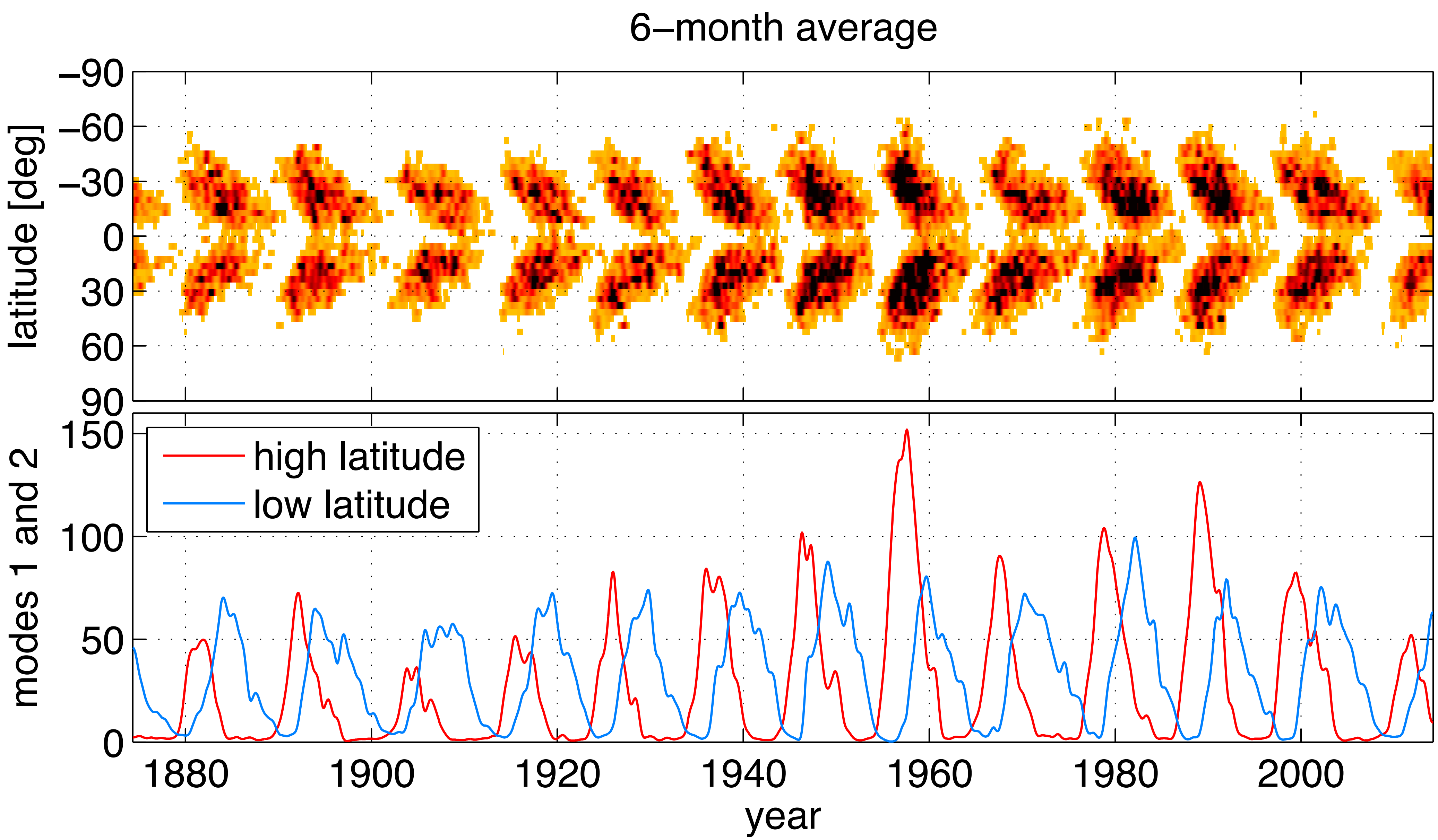
- The number of sunspots **AND** their location are crucial for understanding the variability of the dynamo (and the SSN)





Butterfly diagram

- Most of the dynamics is captured by 2 degrees of freedom
  → "high latitude" mode & "low latitude" mode



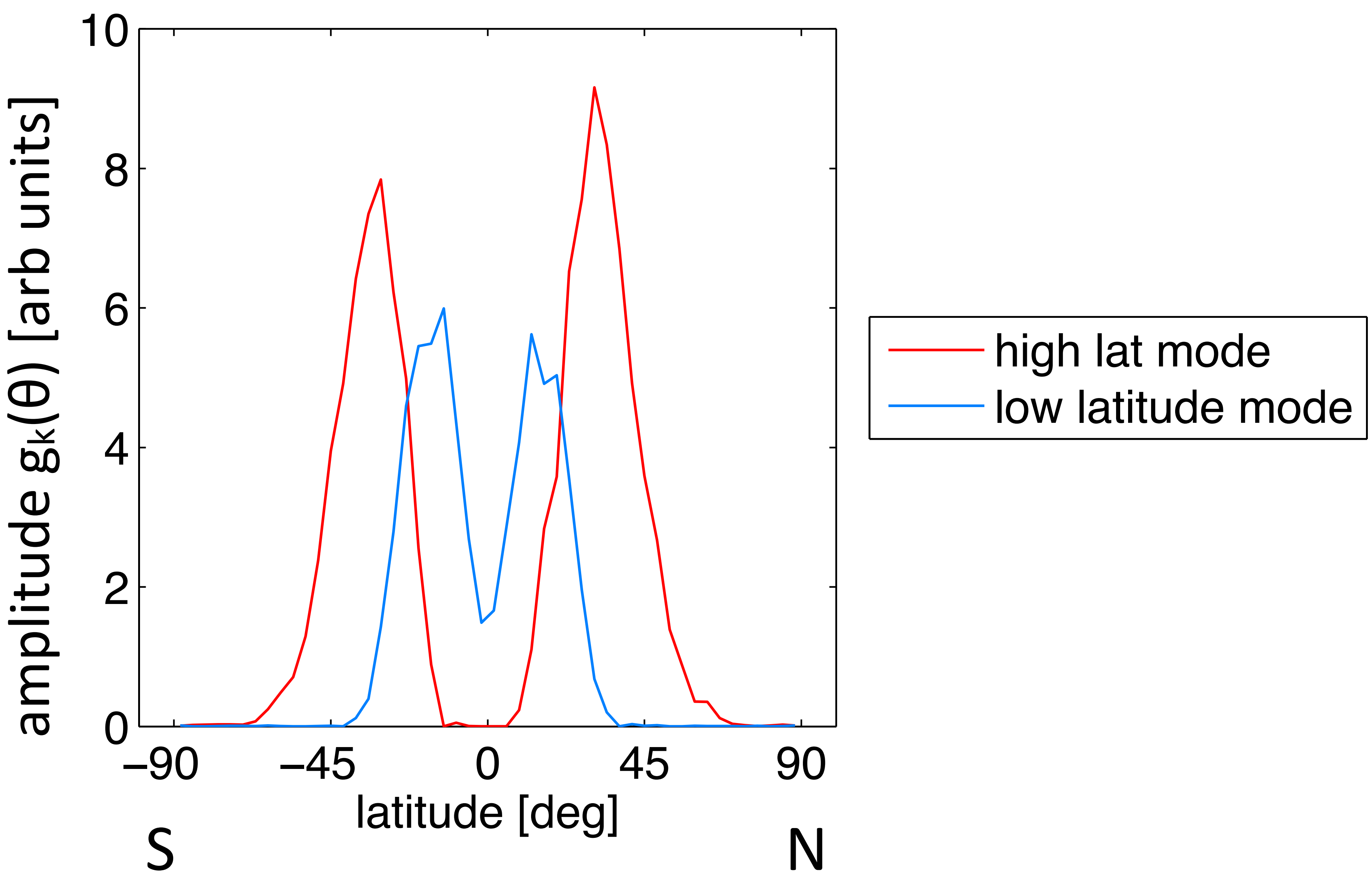
"high latitude" and "low latitude" modes

modes identified by Bayesian positive source separation

Butterfly diagram

- The latitudinal distribution of the 2 modes

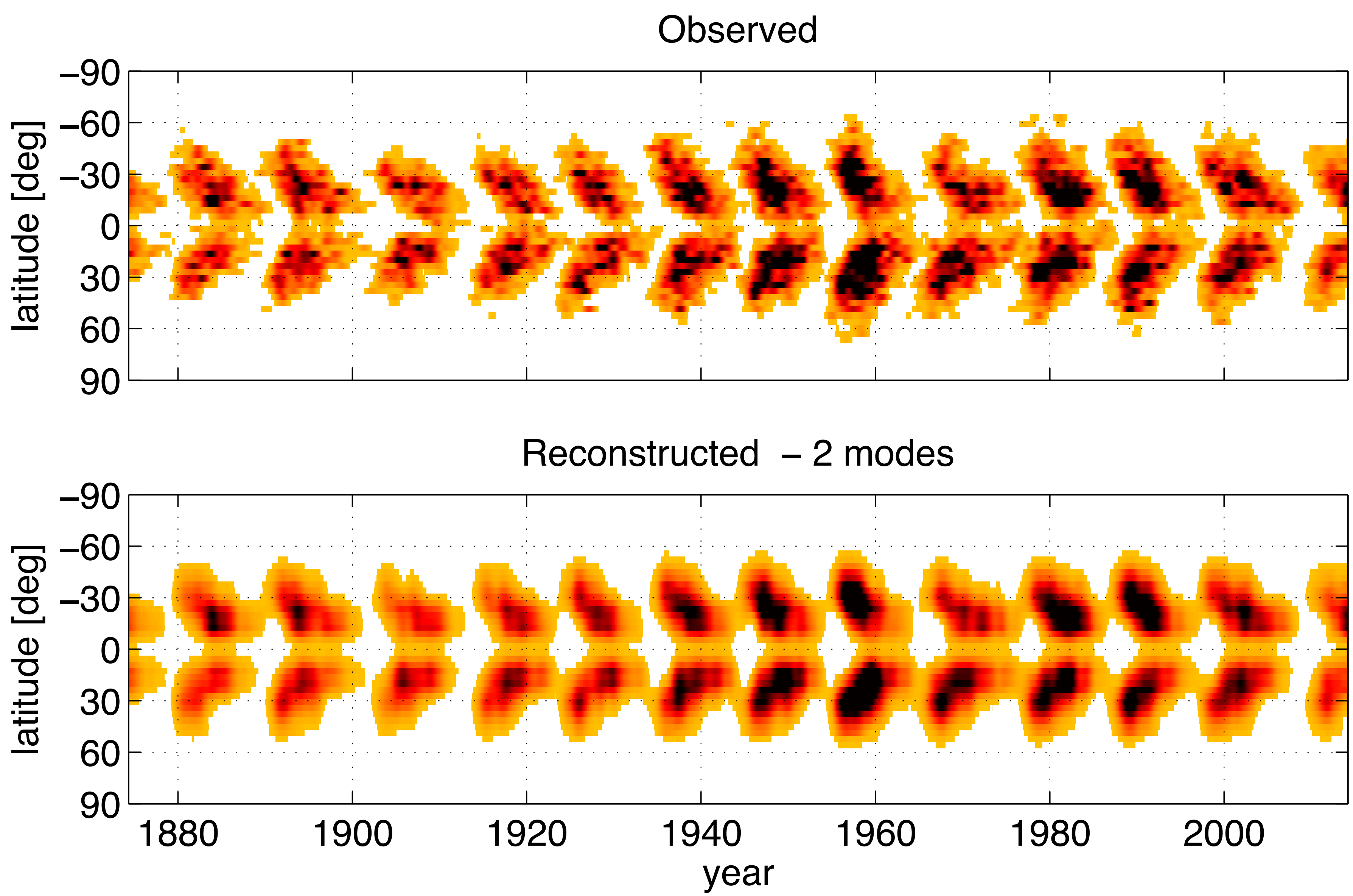
$$\mathrm{Area}(t,\theta) = \sum_{k=1,2} \mathrm{mode}_k(t)\; g_k(\theta)$$



Butterfly diagram

- Sanity check: Reconstruction of the butterfly diagram with 2 modes

$$\text{Area}(t, \theta) = \sum_{k=1,2} \text{mode}_k(t)\; g_k(\theta)$$

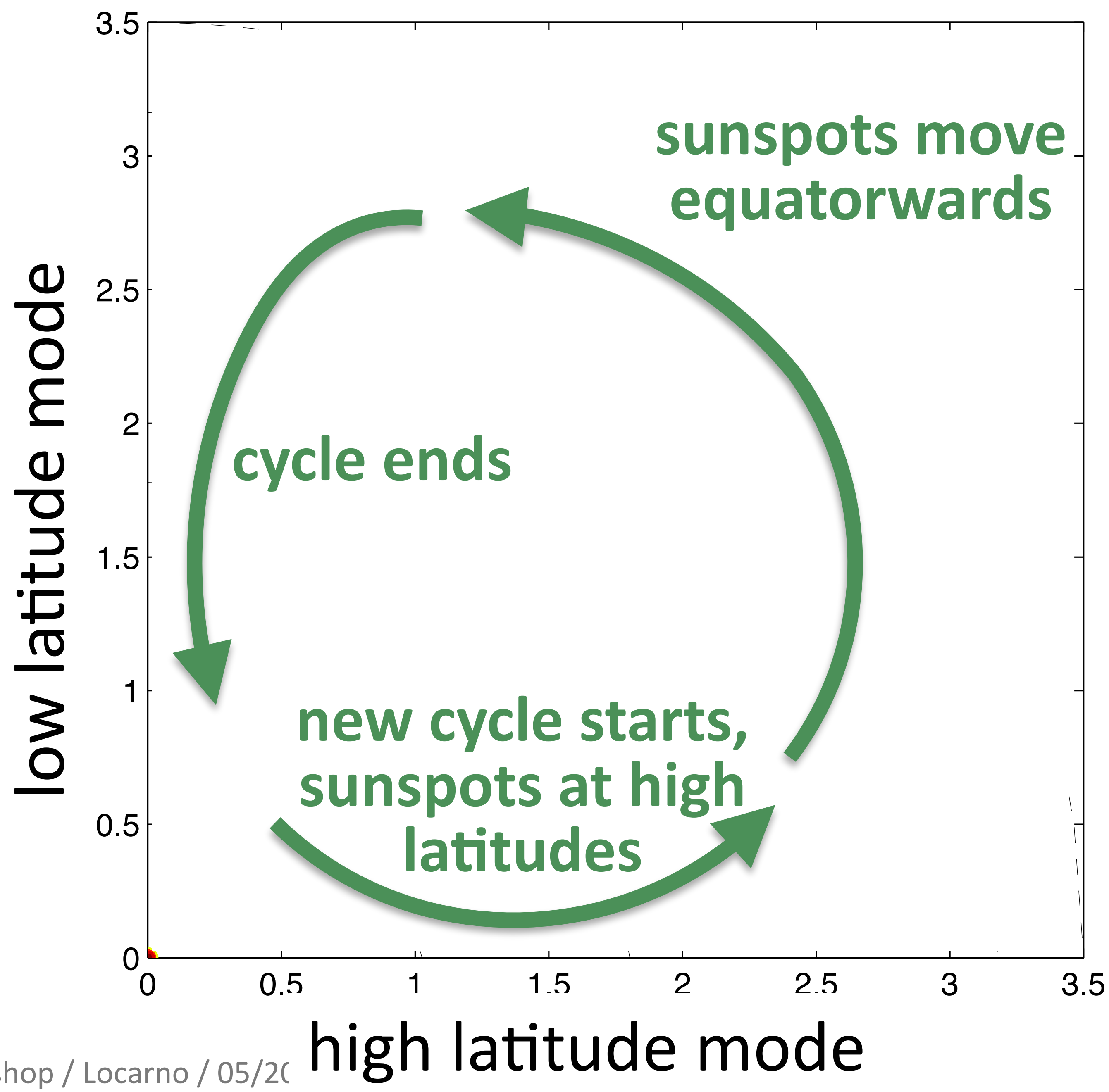


Observed

Reconstructed

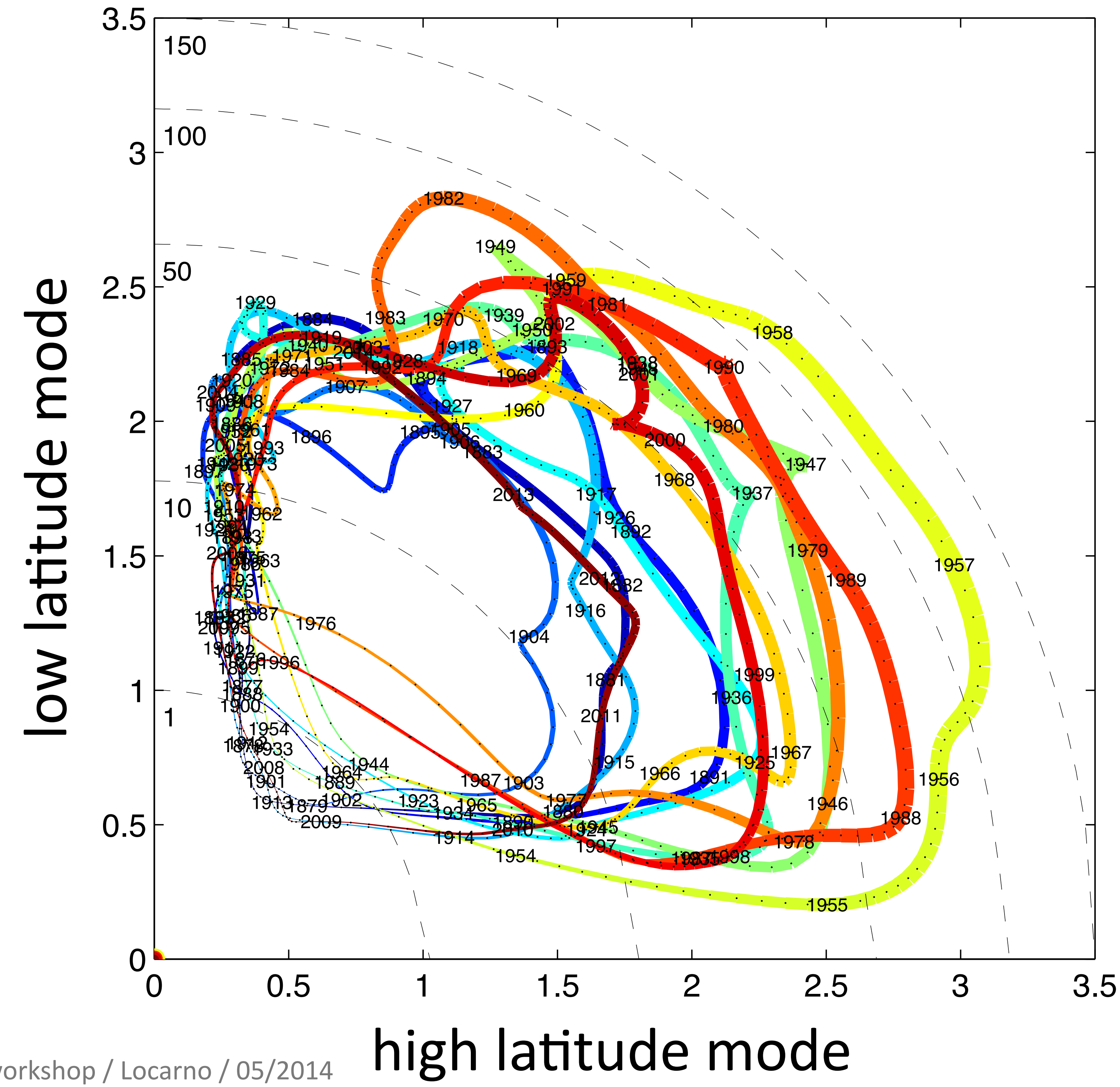
Butterfly diagram

- Plot mode 1 versus mode 2 ("phase space plot") = very condensed representation



Butterfly diagram

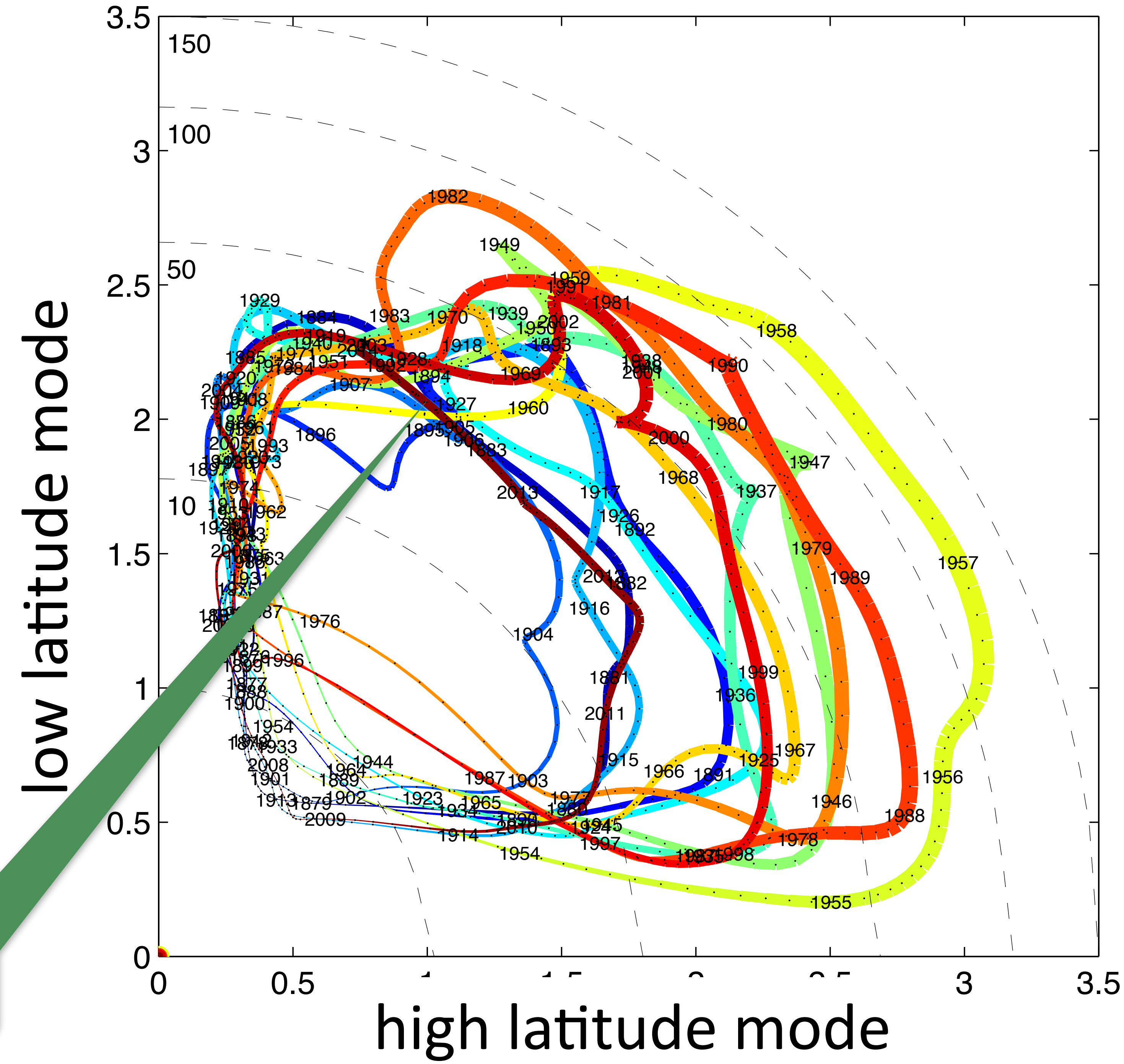
- What the phase space actually looks like



Two solar cycles are similar if their trajectories overlap here

Butterfly diagram

The latest cycle is similar to the one of 1878-1888, not only in SSN, but ALSO in latitudinal distribution

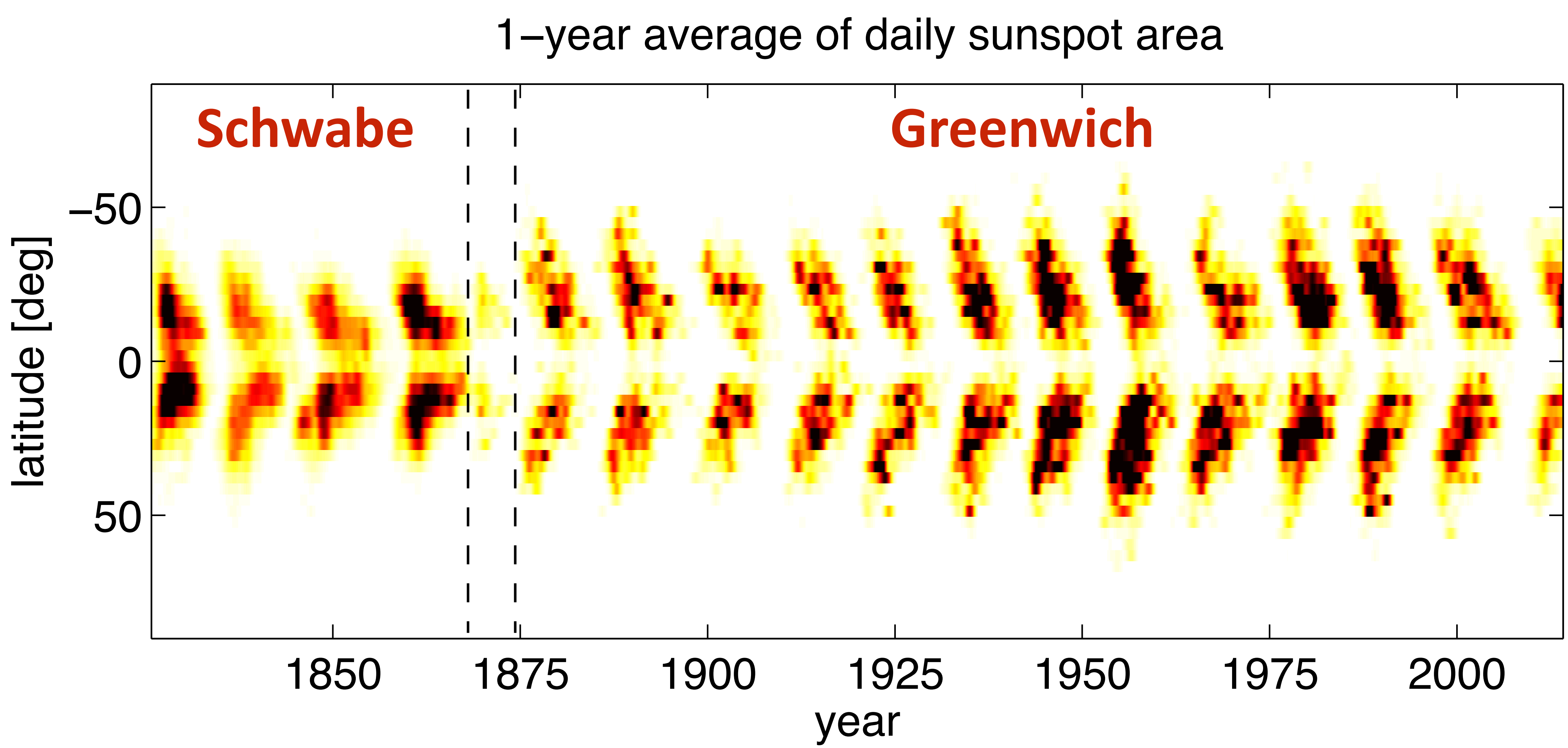


we are here

Sanity check

- Now extend this approach backward in time, using the data from from Schwabe [courtesy Rainer Arlt]

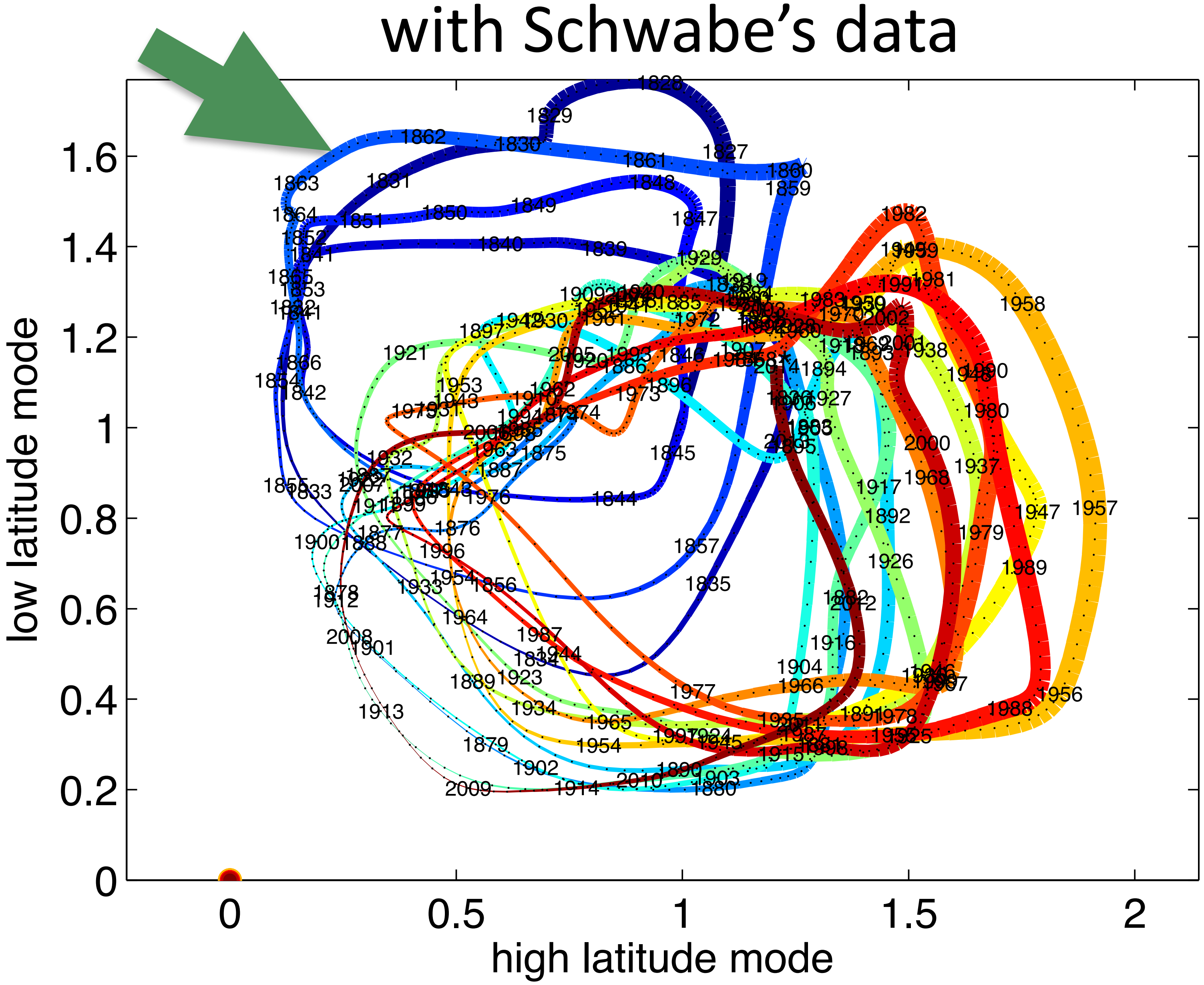
- Butterfly diagram: Greenwich + Schwabe



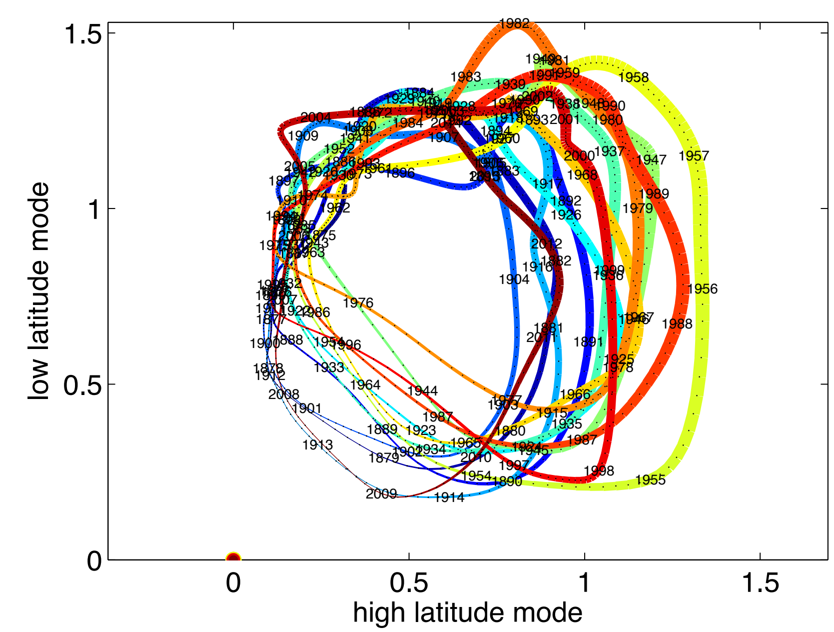
Sanity check

- Schwabe's orbits completely differ from the ones from Greenwich = very unlikely to be due to the Sun

with Schwabe's data



before



Conclusions

- The phase space representation offers **lots of interesting directions to explore**
  - most of the Butterfly diagram captured by just 2 proxies
  - criteria for predicting the shape and amplitude of the solar cycle
  - gives robust criteria for defining the onset of a cycle
  - and much more...

- Reveals biases in the Butterfly diagram, which are not readily visible by eye.
  - Most likely multiple counting of the same active regions

Overall conclusions

- Confidence intervals are essential
  - for doing proper statistics
  - for giving deeper insight into the processing of the data