

Statistical Tests of Uniformity in Solar Activity Indices

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Outline

- Sunspot Number Variability $\alpha \sqrt{R_{I}}$
- RMS Variation in Monthly R_I
- RMS Variation in Daily R_I
- Day-to-Day Variation in Daily R_I
- Group Sunspot Number
- 10.7 cm Radio Flux
- Sunspot Area
- Conclusions

Error Range in Predictions

A decade ago I was using the 2σ uncertainty in the predicted amplitude of the cycle for the error range. This constant value was too small for the monthly values at maximum and too large for the monthly values at minimum.



Errors Proportional to \sqrt{R_{I}}

Since then I have used errors given by 3.3 $\sqrt{R_l}$ which seem to contain ~95% of the variations.



Variations in Monthly R_I

Plotting the RMS variation of the 13 monthly values in the 13-month running mean of R_I since 1749 shows a good fit (albeit with substantial scatter) to 2.1 $\sqrt{R_I}$.



Monthly R₁ Variability Ratio History

Plotting the Monthly Variability Ratio (the ratio of the 13-month RMS variation from the 13-month running mean to the square-root of the13-month running mean itself) as a function of time shows a fairly constant 2.1 with no indication of any long-term trends.



Daily *R*₁ Variability Ratio History

Plotting the Daily Variability Ratio (the ratio of the RMS variation of the daily values from the monthly mean to the square-root of the monthly mean itself) as a function of time also shows a fairly constant value (a somewhat higher 3.0) with no indication of any long-term trends.



Day-to-Day R, Variability Ratio History

Plotting the Day-to-Day Variability Ratio (the ratio of the average absolute difference between a daily value and the value from the day before to the square-root of the monthly mean) as a function of time does show a long-term downward trend.



Group Sunspot Numbers

The Group Sunspot Number, R_G , was devised by Hoyt & Schatten (1998): $R_G = 12.08 \sum k_i G_i / N$ The RGO data was used to normalize this relationship with $k_{RGO} = 1.0$. Plotting smoothed monthly values of R_i vs. G_{RGO} suggests a nonlinear relationship.



Group Sunspot Numbers



There is a weaker dependence on the number of Groups prior to 1947 than afterwards.

After 1946 the number of Groups must be multiplied by an additional 1.2.



Monthly R_G Variability Ratio History

Plotting the R_G Monthly Variability Ratio as a function of time (data after 1995 are derived from USAF Region Reports) shows a fairly constant value from about 1900 to the present but with lower values from ~1850 to ~1900 and higher values from ~1750 to 1810.



Daily *R_G* **Variability Ratio History**

Plotting the R_G Daily Variability Ratio as a function of time shows similar behavior - a fairly constant value from about 1900 to the present but with lower values from ~1850 to 1900 and higher values from ~1750 to 1810..



Day-to-Day R_G Variability Ratio History

Plotting the Day-to-Day Variability Ratio as a function of time does show a long-term downward trend.



10.7 cm Radio Flux

The 10.7 cm radio flux has a nonlinear relationship to the sunspot number. Holland & Vaughn (1984) gave a compound relationship with an exponential for small values and a linear relationship at larger values. This can be approximated by two linear segments.



F10.7 Equivalent

$R_F = (F10.7 - 67.)/0.58, F10.7 < 84$ $R_F = (F10.7 - 56.)/0.94, F10.7 > 84$

Monthly *R_F* Variability Ratio History

Plotting the R_F Monthly Variability Ratio as a function of time shows a similar and fairly constant value.



Daily *R_F* **Variability Ratio History**

Plotting the R_F Daily Variability Ratio as a function of time shows similar behavior.



Day-to-Day R_FVariability Ratio History

Plotting the Day-to-Day Variability Ratio as a function of time shows an initial drop to more constant values.



Area vs. Number

Comparing the RGO areas to the International Sunspot Numbers gives: Area_{RGO} = 16.3 R_I

Comparing the USAF areas to the International Sunspot Numbers gives: Area_{USAF} = 11.2 R_I.

This indicates a scaling factor to bring the USAF areas in line with the RGO areas:

Area_{RGO} = 1.45 Area_{USAF}



Area vs. 10.7 cm Flux

Comparing the RGO areas to the 10.7 cm Radio Flux gives: Area_{RGO} = 18.5*(F10.7 -67.)

Comparing the USAF areas to the 10.7 cm Radio Flux gives: Area_{USAF} = 12.8*(F10.7 – 67.)

This indicates a scaling factor to bring the USAF areas in line with the RGO areas:

Area_{RGO} = 1.44 Area_{USAF}



Which Areas are "Correct"?

Should we use $Area = Area_{RGO} = 1.4 Area_{USAF}$ Or $Area = 0.7 Area_{RGO} = Area_{USAF}$

Sunspot Areas from SOHO







The "flattened" SOHO/MDI intensity images were processed with an IDL procedure that identifies penumbrae and umbrae using break points in the histogram of 8-bit intensities. The total corrected sunspot area for each of the 20,000 images was calculated using the center-to-limb distance of each sunspot pixel.

SOHO Area vs. USAF Area

Comparing the daily sunspot areas from USAF and from SOHO for the time period of May 1996 through December 2010 indicates that it is the USAF data that needs correction.



Sunspot Area Equivalent

$R_A = Area/16.7$

(Where USAF Area is multiplied by 1.4)

Monthly R_A Variability Ratio History

Plotting the R_A Monthly Variability Ratio as a function of time shows a somewhat higher but fairly constant value.



With/Without Correction



The variability ratios for the uncorrected data fall below the average.

Daily *R_A* **Variability Ratio History**

Plotting the R_A Daily Variability Ratio as a function of time shows similar behavior.



Day-to-Day R_AVariability Ratio History

Plotting the Day-to-Day Variability Ratio as a function of time does show a long-term downward trend.



Monthly Variability









Daily Variability



Day-to-Day Variability









Conclusions

- Variations in the Sunspot Number follow Poisson Statistics and are proportional to the square-root of the number itself
- The ratio of the variability in the Sunspot Number to the square-root of the average value is relatively constant in time
- Changes in this ratio may be indicative of scaling changes in the Sunspot Number