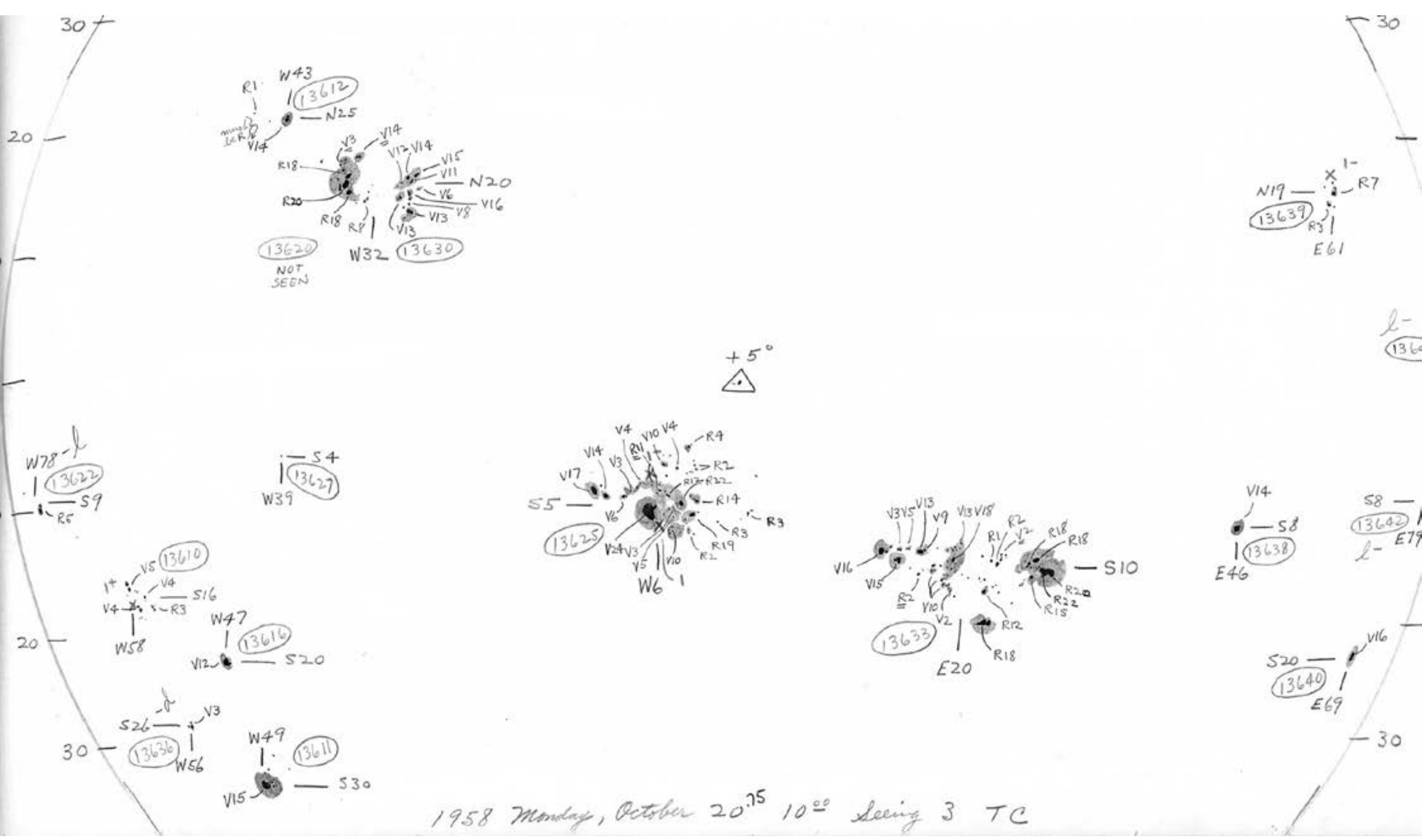
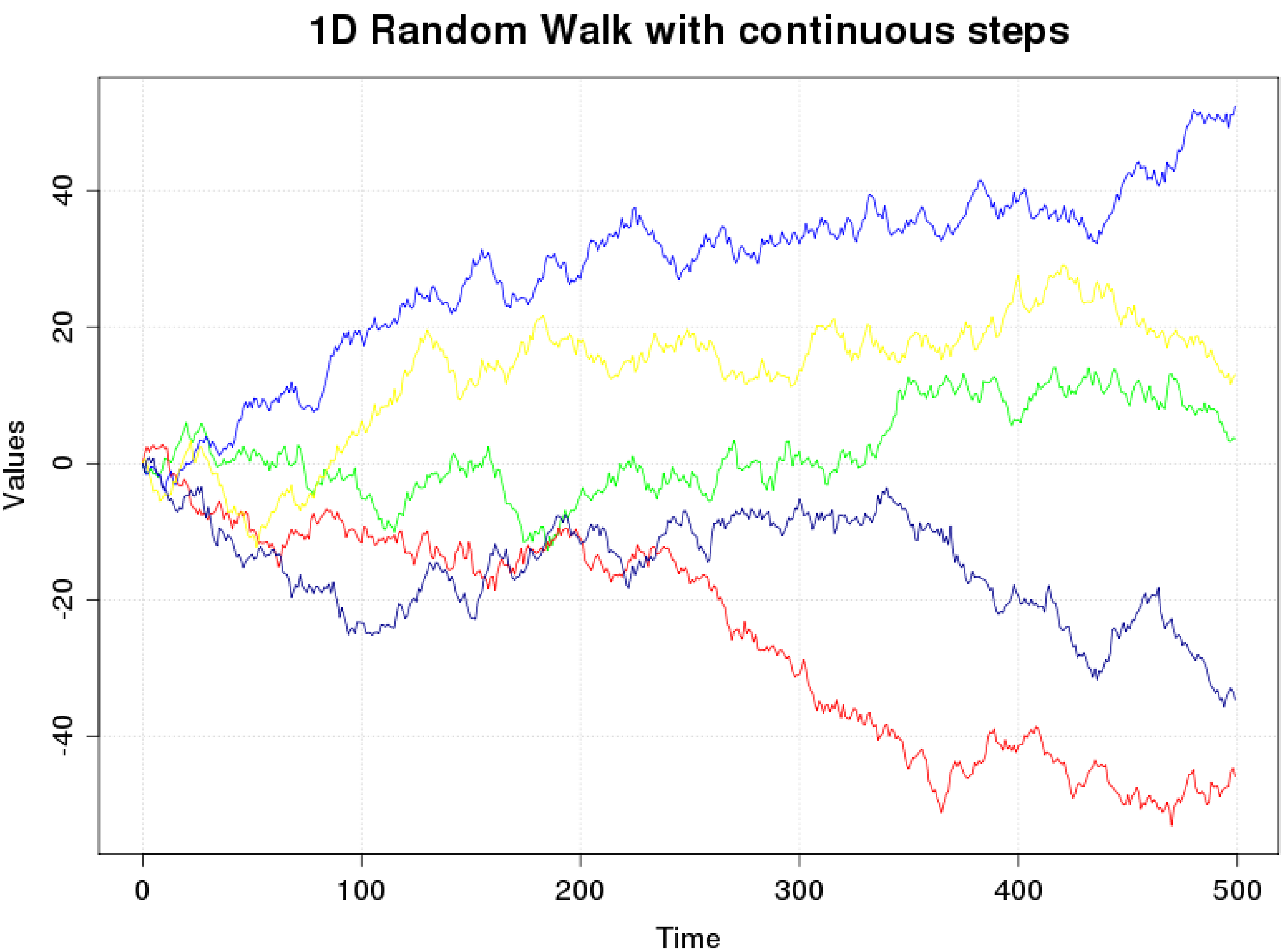
Sunspot random walk and 22-year variation

Jeffrey J. Love & E. Joshua Rigler
USGS Geomagnetism Program

Geophys. Res. Lett., 39, L10103,
doi:10.1029/2012GL051818, 2012.




1D Random Walk with continuous steps
Values
Time
40
20
0
-20
-40
0
100
200
300
400
500

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

This motivates the iterative time-step mapping of magnetic intensity B

$$B_{j-1} \to B_j$$

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

This motivates the iterative time-step mapping of magnetic intensity B

$$B_{j-1} \to B_j$$

For linear kinematic dynamo action, $\mathbf{u} \neq \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}) B_{j-1}$$ gives either marginal steady or polarity oscillations.

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

This motivates the iterative time-step mapping of magnetic intensity B

$$B_{j-1} \to B_j$$

For linear kinematic dynamo action, $\mathbf{u} \neq \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}) B_{j-1} \qquad \text{gives either marginal steady or polarity oscillations.}$$

For nonlinear dynamic dynamo action, $\mathbf{u} = \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}, B_{j-1}) \qquad \text{can give chaos and bias across polarity oscillations.}$$

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

This motivates the iterative time-step mapping of magnetic intensity B

$$B_{j-1} \to B_j$$

For linear kinematic dynamo action, $\mathbf{u} \neq \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}) B_{j-1}$$ gives either marginal steady or polarity oscillations.

For nonlinear dynamic dynamo action, $\mathbf{u} = \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}, B_{j-1})$$ can give chaos and bias across polarity oscillations.

Consider sunspot number R as representing cycle-to-cycle change of B

$$\rho_j(R) = R_j / R_{j-1} \sim \mathcal{P}_R(\rho_j | \sigma^2)$$

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

This motivates the iterative time-step mapping of magnetic intensity B

$$B_{j-1} \rightarrow B_j$$

For linear kinematic dynamo action, $\mathbf{u} \neq \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}) B_{j-1} \qquad \text{gives either marginal steady or polarity oscillations.}$$

For nonlinear dynamic dynamo action, $\mathbf{u} = \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}, B_{j-1}) \qquad \text{can give chaos and bias across polarity oscillations.}$$

Consider sunspot number R as representing cycle-to-cycle change of B

$$\rho_j(R) = R_j / R_{j-1} \sim \mathcal{P}_R(\rho_j | \sigma^2)$$

With a logarithmic transformation,

$$\delta_j = \ln \rho_j \quad \text{and} \quad S_j = \ln R_j$$

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

This motivates the iterative time-step mapping of magnetic intensity B

$$B_{j-1} \to B_j$$

For linear kinematic dynamo action, $\mathbf{u} \neq \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}) B_{j-1} \qquad \text{gives either marginal steady or polarity oscillations.}$$

For nonlinear dynamic dynamo action, $\mathbf{u} = \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}, B_{j-1}) \qquad \text{can give chaos and bias across polarity oscillations.}$$

Consider sunspot number R as representing cycle-to-cycle change of B

$$\rho_j(R) = R_j / R_{j-1} \sim \mathcal{P}_R(\rho_j | \sigma^2)$$

With a logarithmic transformation,

$$\delta_j = \ln \rho_j \quad \text{and} \quad S_j = \ln R_j$$

With this, we now consider differences, rather than ratios

$$\delta_j(S) = S_j - S_{j-1} \sim \mathcal{P}_S(\delta_j | \sigma^2)$$

Cycle-to-Cycle Log-Normal Random Walk

Induction equation describes time dependence of solar magnetic field $\mathbf{B}$

$$\partial_t \mathbf{B} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

This motivates the iterative time-step mapping of magnetic intensity B

$$B_{j-1} \rightarrow B_j$$

For linear kinematic dynamo action, $\mathbf{u} \neq \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}) B_{j-1} \qquad \text{gives either marginal steady or polarity oscillations.}$$

For nonlinear dynamic dynamo action, $\mathbf{u} = \mathbf{u}(\mathbf{B}, t)$,

$$B_j = f(\eta, \mathbf{u}, B_{j-1}) \qquad \text{can give chaos and bias across polarity oscillations.}$$

Consider sunspot number R as representing cycle-to-cycle change of B

$$\rho_j(R) = R_j / R_{j-1} \sim \mathcal{P}_R(\rho_j | \sigma^2)$$

With a logarithmic transformation,

$$\delta_j = \ln \rho_j \quad \text{and} \quad S_j = \ln R_j$$

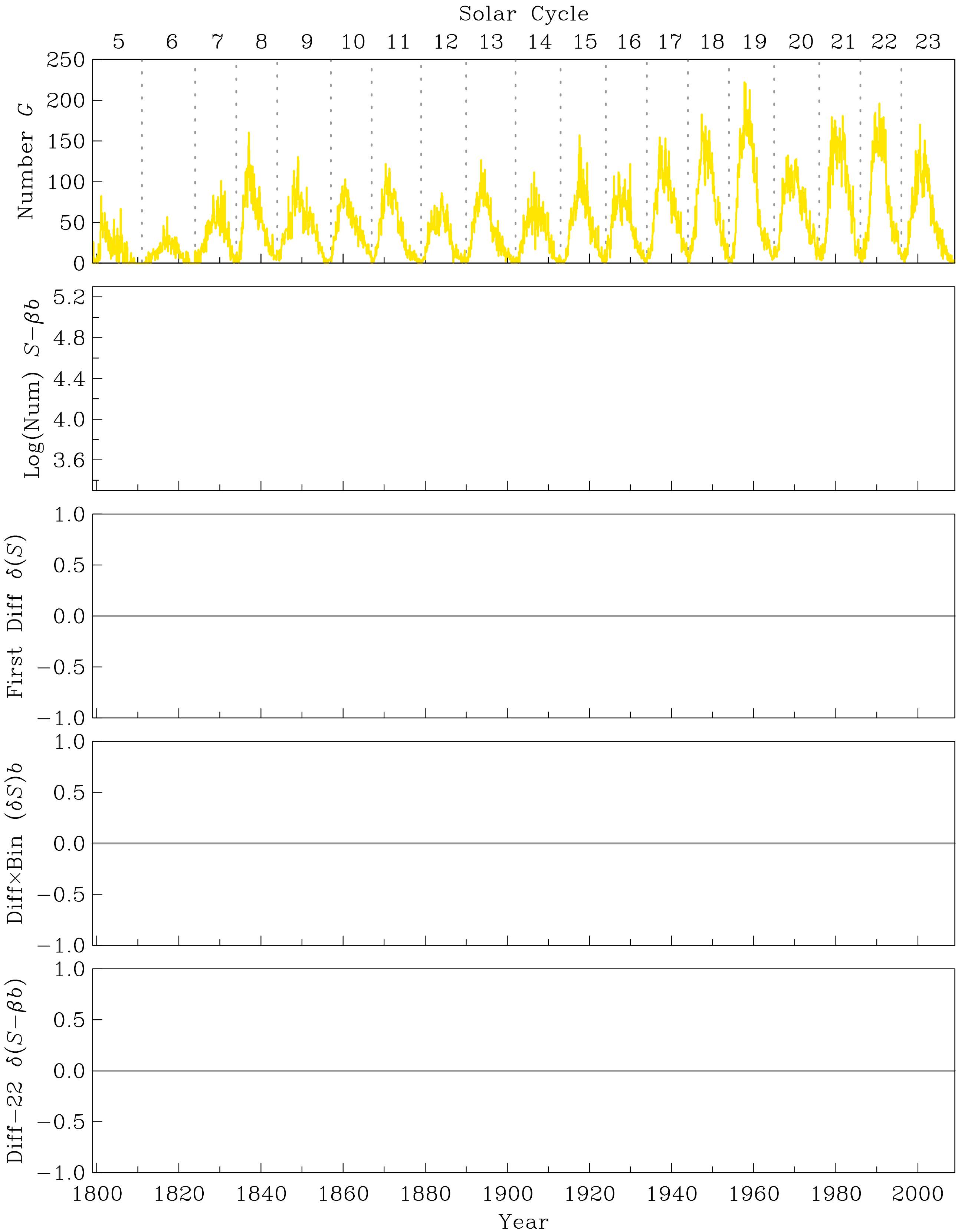
With this, we now consider differences, rather than ratios

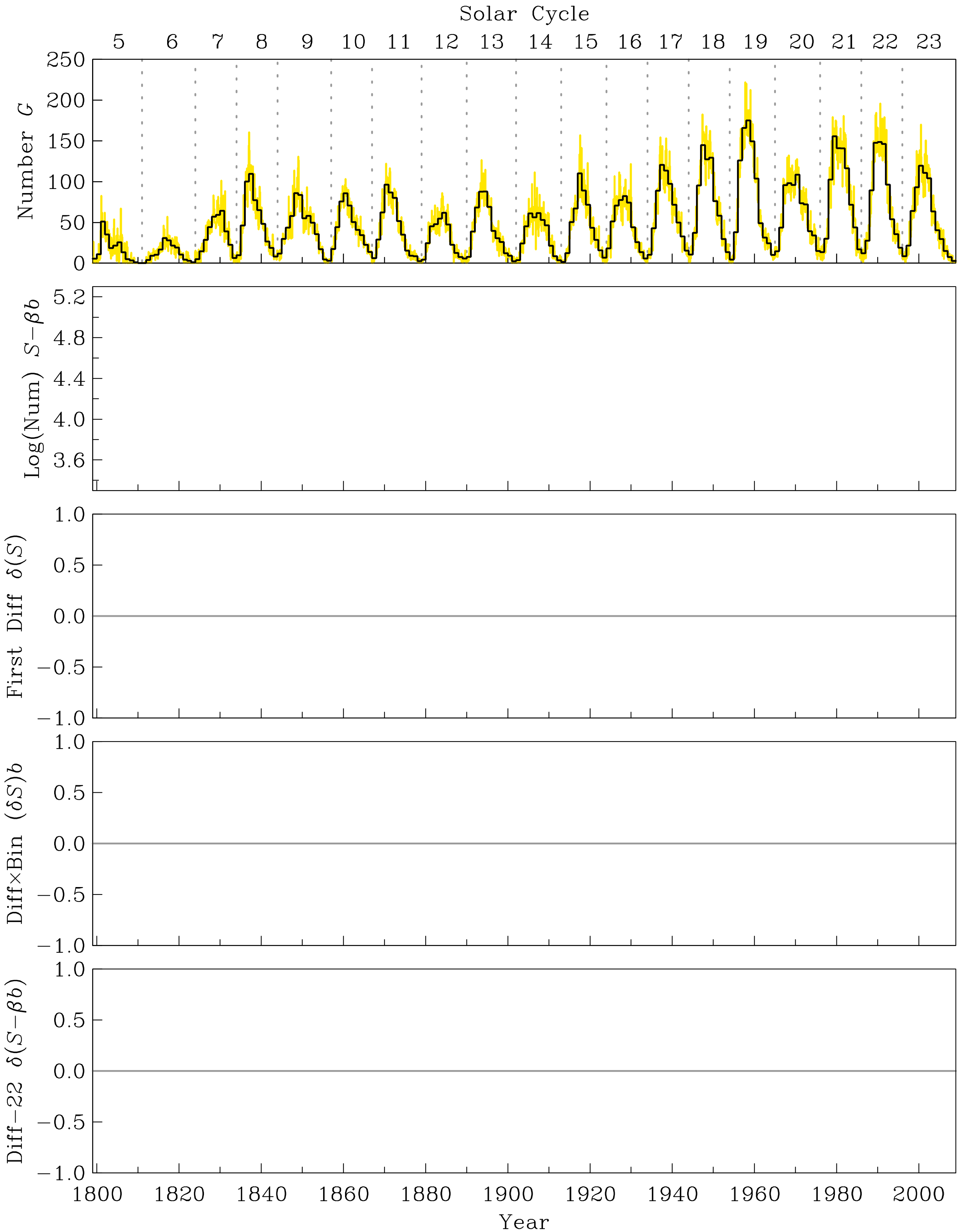
$$\delta_j(S) = S_j - S_{j-1} \sim \mathcal{P}_S(\delta_j | \sigma^2)$$

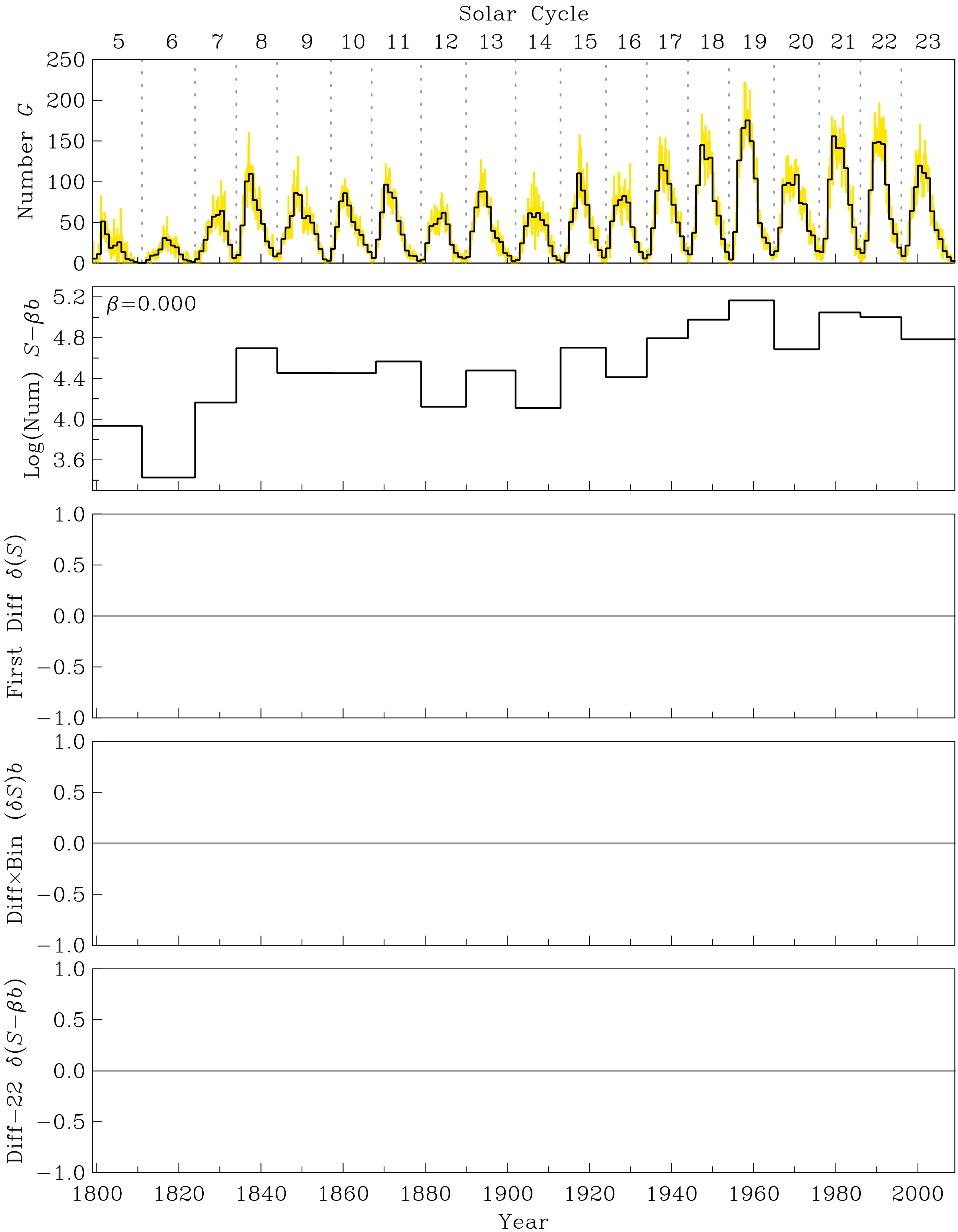
As a null hypothesis we test, for possible rejection, a zero-mean normal model

$$\mathcal{P}_S(\delta_j | \sigma^2) = \mathcal{N}_S(\delta_j | \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\delta_j^2}{2\sigma^2}\right]$$

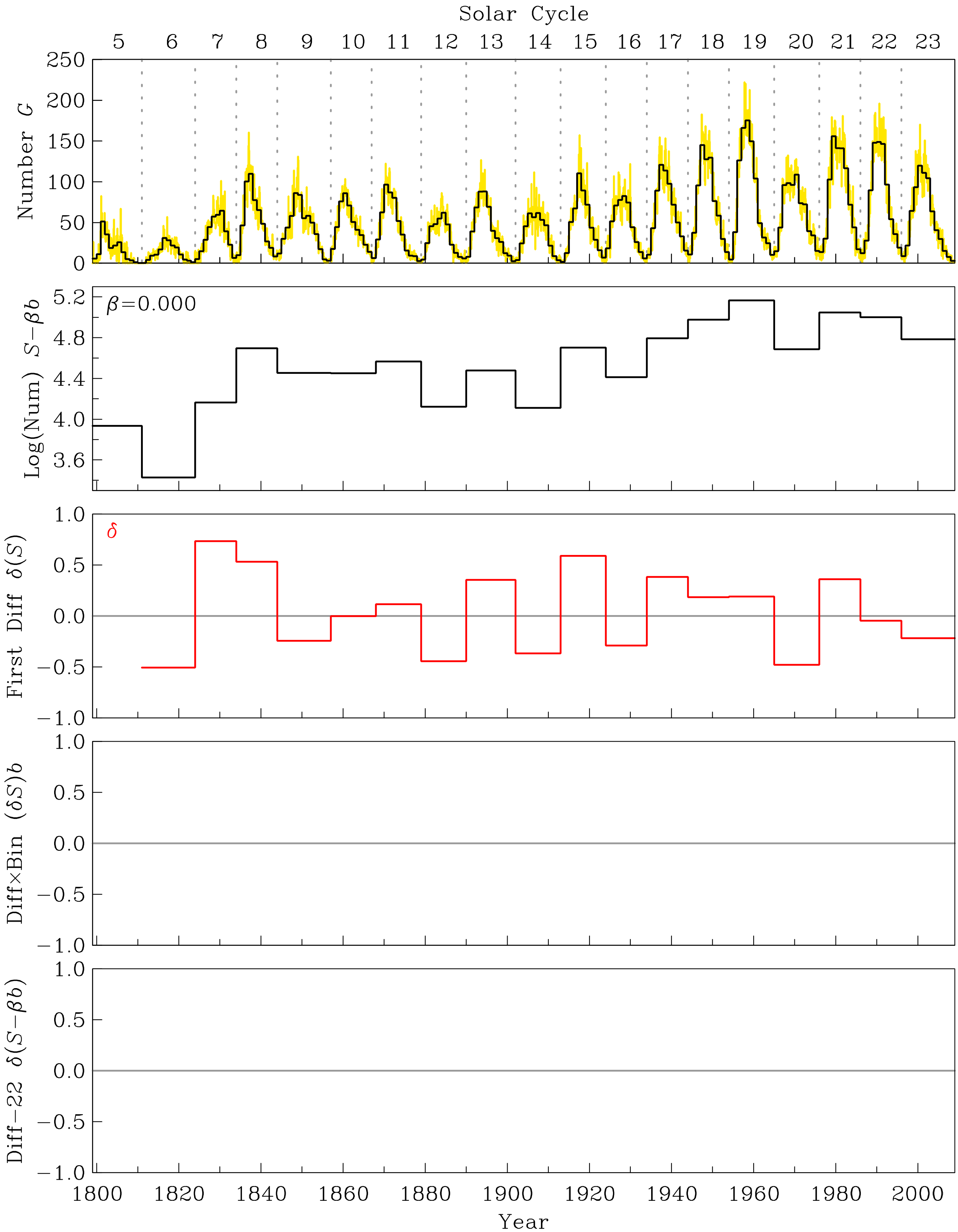
Upon transformation back to non-log space, we have a log-normal random walk.


Solar Cycle
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number G
Log(Num) $S-\beta b$
First Diff $\delta(S)$
Diff×Bin $(\delta S)b$
Diff−22 $\delta(S-\beta b)$
1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000
Year

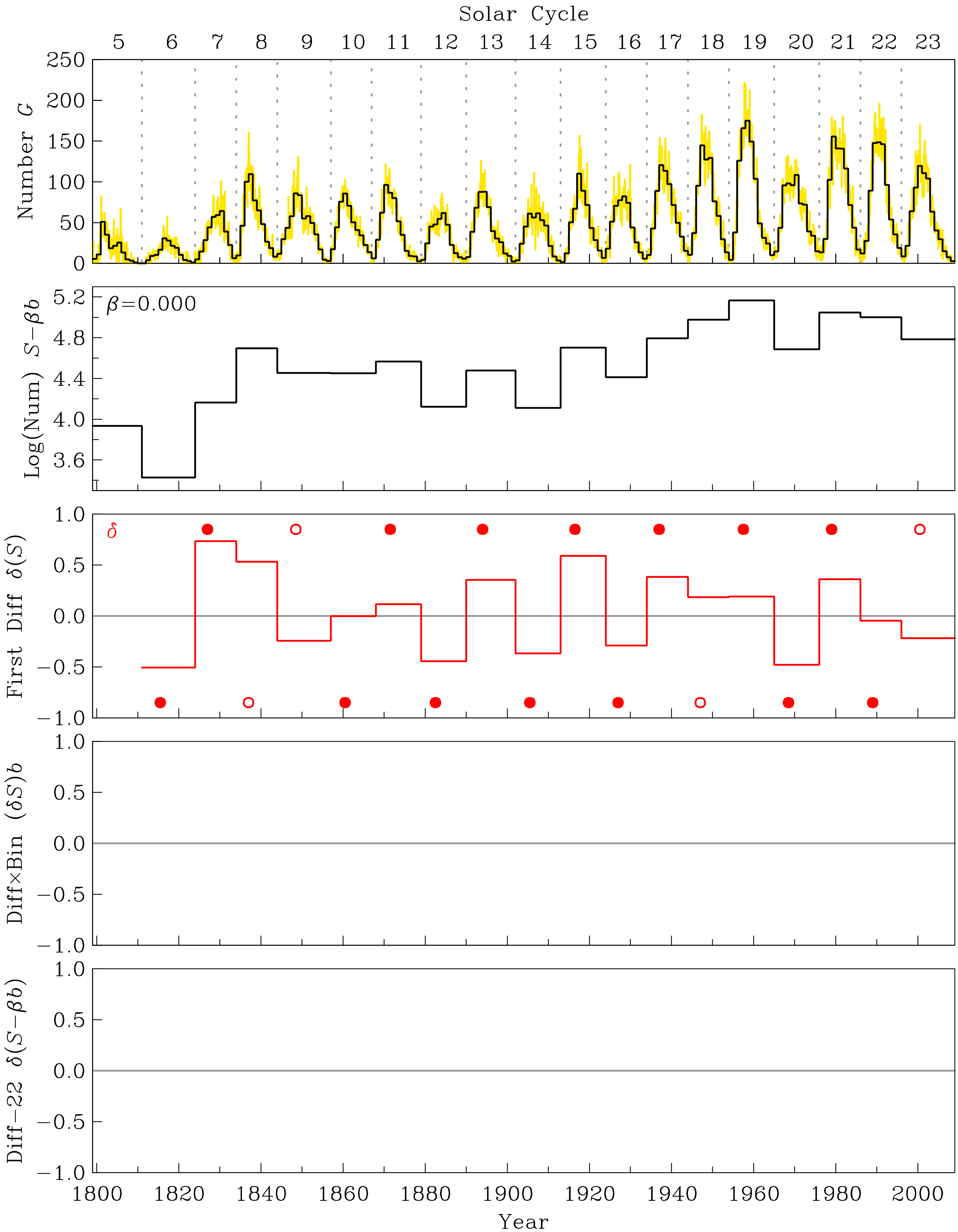

Solar Cycle
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number G
0 50 100 150 200 250
Log(Num) $S-\beta b$
3.6 4.0 4.4 4.8 5.2
First Diff $\delta(S)$
Diff×Bin $(\delta S)b$
Diff−22 $\delta(S-\beta b)$
−1.0 −0.5 0.0 0.5 1.0
1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000
Year



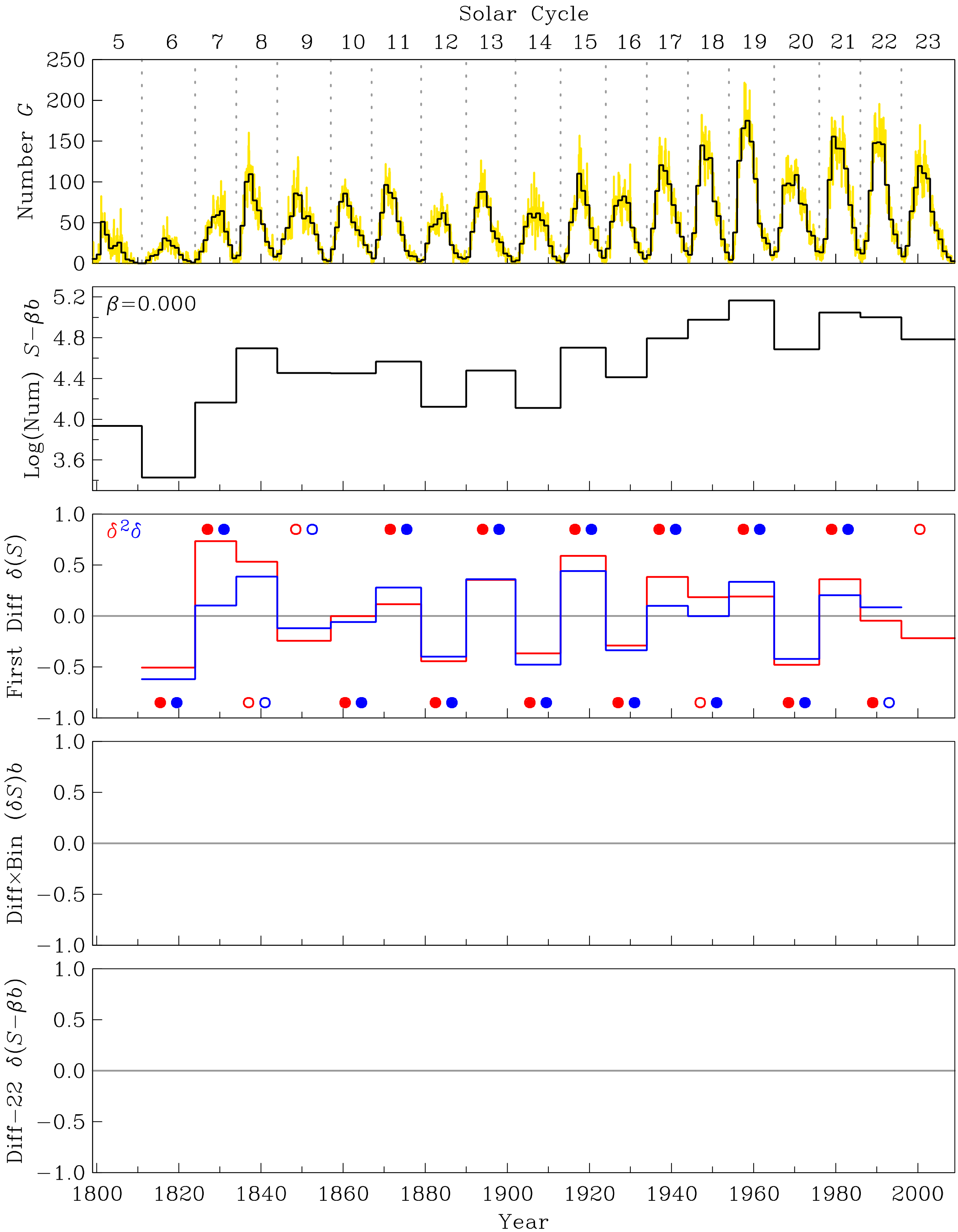
Solar Cycle
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
Number G
250
200
150
100
50
0
Log(Num) S−βb
β=0.000
5.2
4.8
4.4
4.0
3.6
First Diff δ(S)
1.0
0.5
0.0
−0.5
−1.0
Diff×Bin (δS)b
1.0
0.5
0.0
−0.5
−1.0
Diff−22 δ(S−βb)
1.0
0.5
0.0
−0.5
−1.0
1800
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000
Year



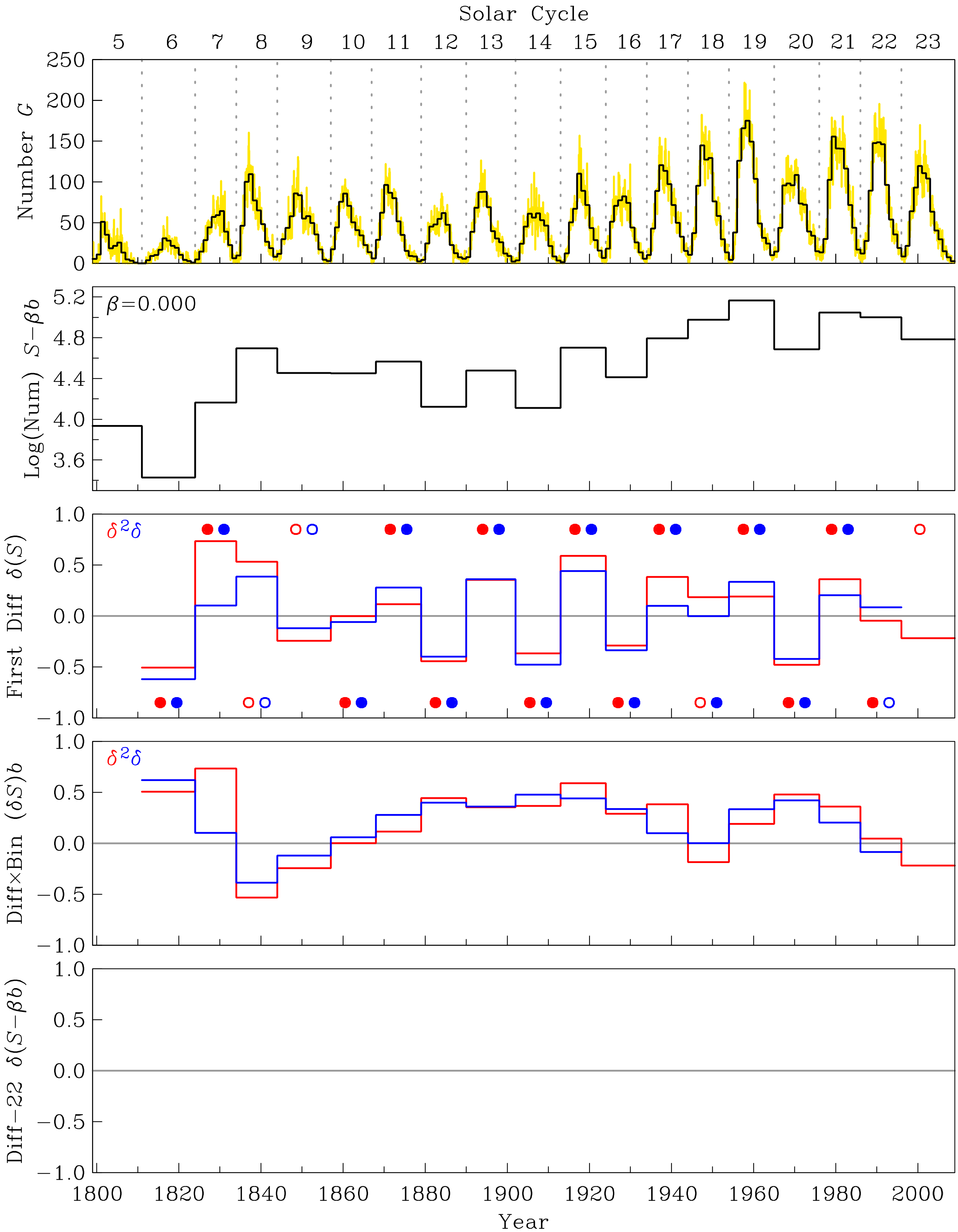
Solar Cycle
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number G
Log(Num) S−βb
β=0.000
First Diff δ(S)
δ
Diff×Bin (δS)b
Diff−22 δ(S−βb)
1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000
Year



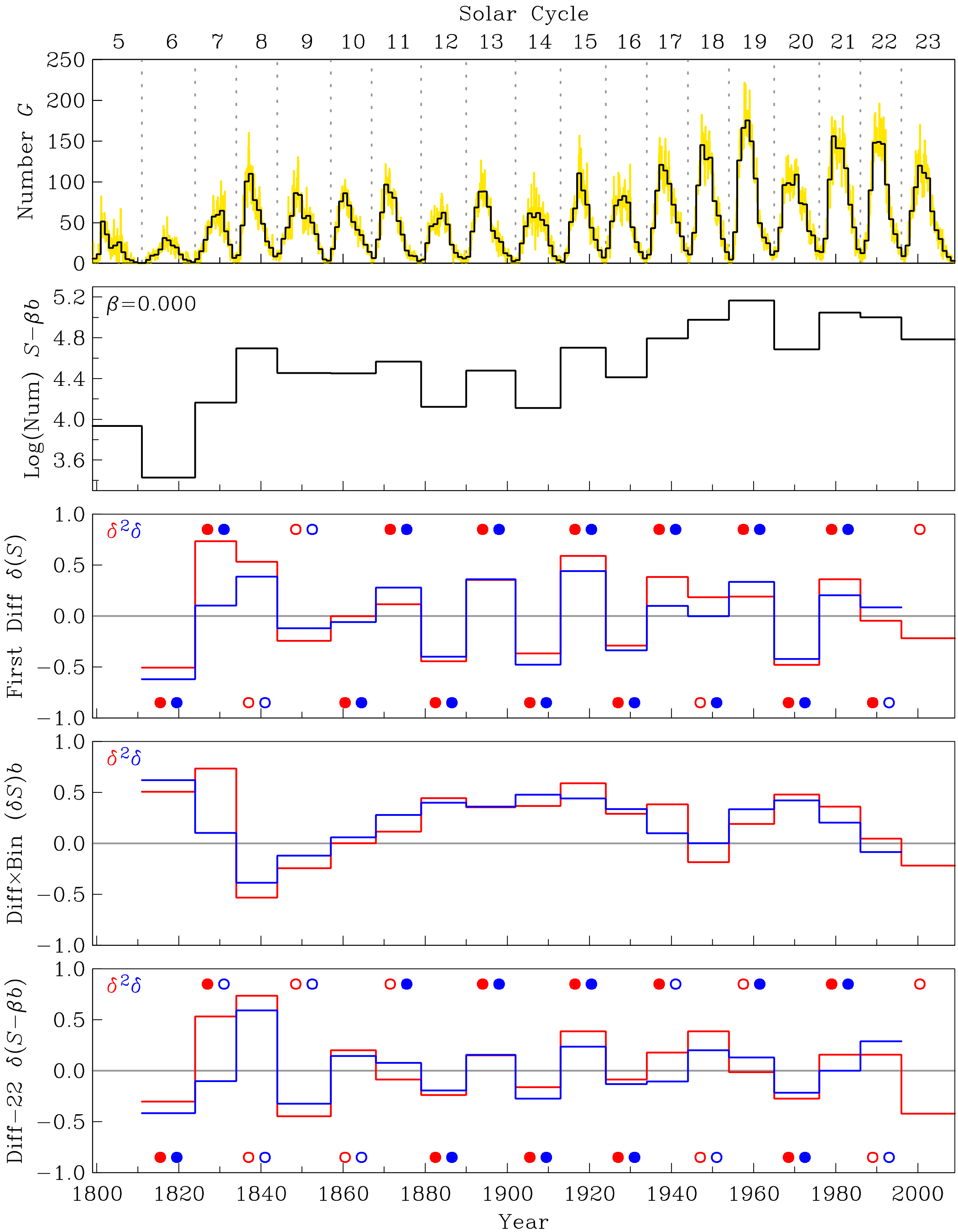
Solar Cycle
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number G
Log(Num) S−βb
β=0.000
First Diff δ(S)
δ
Diff×Bin (δS)b
Diff−22 δ(S−βb)
1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000
Year



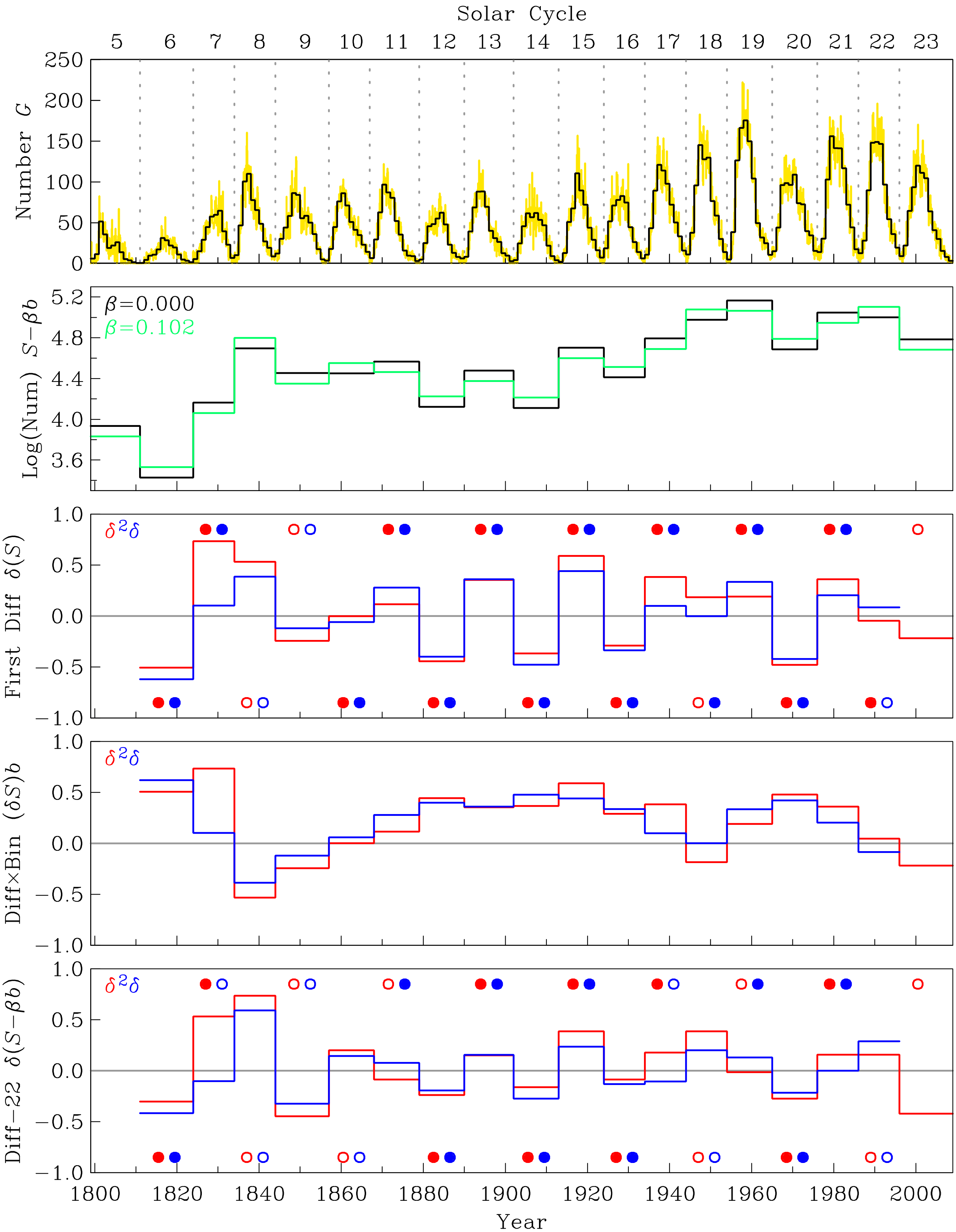
Solar Cycle
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number G
Log(Num) $S-\beta b$
β=0.000
First Diff $\delta(S)$
$\delta^2\delta$
Diff×Bin $(\delta S)b$
Diff−22 $\delta(S-\beta b)$
Year



Solar Cycle
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number G
β=0.000
Log(Num) $S-\beta b$
$\delta^2\delta$
First Diff $\delta(S)$
$\delta^2\delta$
Diff×Bin $(\delta S)b$
Diff−22 $\delta(S-\beta b)$
1800 1820 1840 1860 1880 1900 1920 1940 1960 1980 2000
Year



Solar Cycle
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
Number G
Log(Num) S−βb
β=0.000
First Diff δ(S)
δ²δ
Diff×Bin (δS)b
Diff−22 δ(S−βb)
Year



Solar Cycle
5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
Number G
Log(Num) S−βb
β=0.000
β=0.102
First Diff δ(S)
δ²δ
Diff×Bin (δS)b
δ²δ
Diff−22 δ(S−βb)
δ²δ
Year

Retrospective statistical tests for sunspot number since 1800:

- No 22-yr variation for log-max SSN can be rejected, $p < 0.01$, but for log-sums $p = 0.38$.

- Random ($p = 0.29$) log-normal ($p=0.67$) cycle-to-cycle change minus 22-yr cannot be rejected.

- Log-normal random walk as a description of long-term trend cannot be rejected, $p = 0.27$.

Prediction:

- Probability of descent to Dalton-like minimum before cycle 26 is 0.02.