THE SUN 360

Solar Cycle Predictions (Invited Review)

W. Dean Pesnell

Abstract Solar cycle predictions are needed to plan long-term space missions, just as weather predictions are needed to plan the launch. Fleets of satellites circle the Earth collecting many types of science data, protecting astronauts, and relaying information. All of these satellites are sensitive at some level to solar cycle effects. Predictions of drag on low-Earth orbit spacecraft are one of the most important. Launching a satellite with less propellant can mean a higher orbit, but unanticipated solar activity and increased drag can make that a Pyrrhic victory as the reduced propellant load is consumed more rapidly. Energetic events at the Sun can produce crippling radiation storms that endanger all assets in space. Solar cycle predictions also anticipate the shortwave emissions that cause degradation of solar panels. Testing solar dynamo theories by quantitative predictions of what will happen in 5 – 20 years is the next arena for solar cycle predictions. A summary and analysis of 75 predictions of the amplitude of the upcoming Solar Cycle 24 is presented. The current state of solar cycle predictions and some anticipations of how those predictions could be made more accurate in the future are discussed.

Keywords Solar cycle · Predictions

1. Introduction

Our knowledge of the production, transport, and destruction of the solar magnetic field by the solar dynamo is tested by solar cycle predictions. Models of the dynamo are validated by their ability to predict solar activity over short and long timescales. Predictions of the magnitude and timing of Solar Cycle 24 are also used by a variety of space weather groups to estimate orbital drag and other consequences of space weather in the upcoming cycle. Space weather operators use solar activity predictions to plan when to reboost satellites

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Figure 1 Monthly and annual sunspot numbers for the decline of Solar Cycle 23 (SC23) and the onset of Solar Cycle 24 (SC24). Black symbols refer to SC24 and blue to SC23. The larger symbols are the annual averages. A predicted amplitude for SC24 is shown, based on the SODA index and the analysis of Pesnell (2009, 2011). The rise to the maximum of SC24 is well fit by the curve, which predicts SC24 will peak at an amplitude of 82 ± 20 in the middle of 2013 (2013.5). The epoch of solar minimum was set to December 2008.
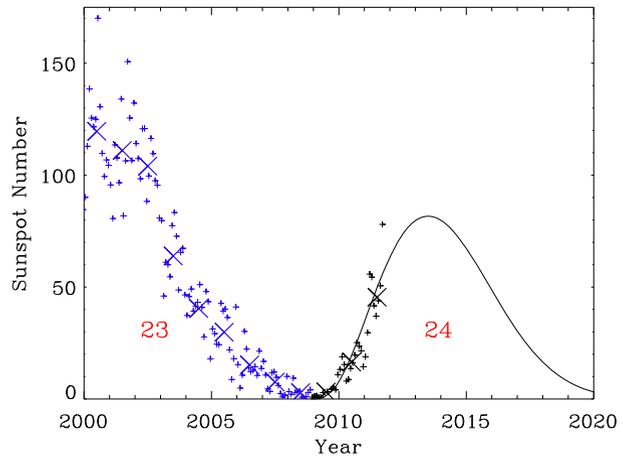
in low Earth orbit (LEO), to anticipate radiation exposure for current and upcoming missions, and to plan for outages in radio-based communication and navigation systems. Space weather operators also want to know the significance of each prediction compared to other predictions.

We discuss here 75 predictions of the amplitude of Solar Cycle 24, emphasizing the predictions that have appeared since Pesnell (2008). Many of them were published before solar minimum and represent our efforts to anticipate solar maximum at ever-earlier epochs. The predictions are analyzed within categories to determine trends and consistencies. We also analyze several indices to show that the solar cycle has some predictability. Finally, the number of spotless days is examined as a possible precursor of solar activity in an upcoming solar cycle.

The amplitude of the annual averaged sunspot number for Solar Cycle n will be called R_n. At the time of writing this article, the annual-averaged sunspot number for 2011 is 45 and still increasing (Figure 1).

2. What Should Be Predicted?

Predicting the sunspot number (R_Z) began when a cyclical behavior was noticed by Schwabe (1844). The sunspot number then played an important role in developing several forecasting techniques in the twentieth century. One example is the autoregressive algorithm developed by Yule (1927) to understand and predict R_Z. These autoregressive moving average (ARMA) techniques have become extremely popular in time series analysis (Lo and Mackinlay, 2002). Mandelbrot and Wallis (1969) also used R_Z as a test series in their early work in chaos dynamics. These algorithms required data series from nature that were not perfectly periodic to study their forecasting ability. This trend continued when the study of solar activity and its effects on satellites became important. Thus, as a result of its availability and variability, sunspot number is the most commonly predicted solar activity index.

Although R_Z has a long history of prediction, it has several difficulties that reduce the quality of solar cycle predictions that use only R_Z. There is a poor understanding of how R_Z is connected to the solar dynamo and how to convert the output of dynamo models into R_Z. Other quantities are more direct measures of the energy released by solar activity. Some aspects of the solar magnetic field, such as coronal holes and the high-speed streams emitted

therefrom, do not track R_Z and require an independent prediction. Another example is the separate evolution of the northern and southern hemispheres. Known as the north–south asymmetry, this difference was accentuated in Solar Cycle 23 when the south peaked two years after the north and continues with the slower buildup of active regions in the south during the rise of Solar Cycle 24. Cosmic rays are a significant source of radiation hazard in space whose flux is anti-correlated with the sunspot number. Geomagnetic activity has one component that is proportional to R_Z and another, which can be a source of significant space weather, that resembles the sunspot number but is delayed by several years.

Even when the appropriate physical variables are used, the consumers of solar activity predictions have their preferred variables. One popular substitute for R_Z is $F_{10.7}$, the solar spectral irradiance at a wavelength of 10.7 cm (2.8 GHz). One advantage of $F_{10.7}$ is its almost immediate availability. $F_{10.7}$ is released within hours of its measurement while the definitive R_Z is delayed by a month. There is also a wide range of timescales for predictions (from seconds to decades) as well as a wide range of user requirements for data latency and sensitivity to false alarms.

Sunspots are produced by the solar magnetic field, and that field would be a better quantity to predict. A complete model of the solar dynamo would produce additional information, such as the distribution of magnetic field by hemisphere and with time, to compare with observations. Along with predictions would come explanations of the butterfly diagram, active longitudes, and other empirical relationships that exist in the sunspot record. Unfortunately, the length of time with quantitative magnetic field data is quite short compared to the sunspot number record. This limits the sampling to the last five sunspot cycles and means the magnetic field data covers only the above average cycles from $20-23$. If the goal becomes predicting the convection zone dynamics and solar dynamo, then the available data covers only Solar Cycle 23 and the beginning of Solar Cycle 24.

Relating the calculated response of an internal dynamo to the external observations is another issue. What is the metric for comparing the theoretical calculations of the magnetic field with how we observe that field at and above the solar surface?

However, R_Z is a proxy for many other indexes, and it merges onto the radionuclide record, allowing these other indices to be extended back thousands of years. The occurrence rate of, and energy released by, solar flares is well correlated with the sunspot number, as is the rate of coronal mass ejections. In general, the sunspot number (or a proxy index such as $F_{10.7}$) will remain the basic quantity reported in solar cycle predictions.

3. How Well Have We Done Before?

Even though R_Z has been used to test prediction algorithms, the need to publish solar cycle predictions became important when we began putting assets in space that are directly affected by solar activity over their lifetimes of ten or more years. Starting with Solar Cycle 20 we have a trail of amplitude and timing forecasts of upcoming solar cycles. Along with the forecasts came retrospective studies of how well those forecasts agreed with the actual amplitude.

A large variety of predictions were made of the amplitude and timing of Solar Cycles 20 through 24. King-Hele (1963) and King-Hele (1966) discussed predictions of Solar Cycle 20. Ohl (1966) introduced the geomagnetic precursor for a prediction of Solar Cycle 20.

Sargent (1978) discussed the early rise of Solar Cycle 21 in the context of 15 predictions. Ohl and Ohl (1979) continued developing the geomagnetic precursor, using a geomagnetic precursor pair to produce a prediction of Solar Cycle 21 before solar minimum by removing

the current cycle's activity. Schatten *et al.* (1978) introduced the use of the magnetic conditions in the solar polar regions as precursors of future activity with a set of predictions of the amplitude of Solar Cycle 21. McIntosh *et al.* (1979) and Brown (1986) reported on the working group that was convened to consider the predictions of Solar Cycles 21 and 22, the former before the peak of Solar Cycle 21 and the latter after. Brown (1986) emphasized that the statistical methods did a poorer job than the precursors of predicting Solar Cycle 21.

Withbroe (1989) took a retrospective look at the predictions of Solar Cycle 21, showing that the precursor methods were more accurate than the statistical methods (roughly the current climatological category). He then discussed the predictions of Solar Cycle 22 during the rise to maximum, while Brown (1992) discussed the predictions of Solar Cycle 22 in retrospect. Once again the precursor predictions were a more reliable category. Li, Yun, and Gu (2001) reached a similar conclusion.

Based on these results, geomagnetic precursors were an important part of the consensus prediction for Solar Cycle 23 (Joselyn *et al.*, 1997). However, all of the predictions for Solar Cycle 23 from geomagnetic precursor methods, as evaluated by Hathaway, Wilson, and Reichman (1999), indicated a larger amplitude than was observed, although the actual amplitude was within or just outside their 2σ error estimates.

The Solar Cycle 24 Prediction Panel also decided to rely on precursors, both solar and geomagnetic, as an important component of their consensus prediction (Biesecker and the Solar Cycle 24 Prediction Panel, 2007).

4. Hurst Exponent of Solar and Geomagnetic Indices

Can the sunspot number be predicted several years in advance with any degree of confidence? This is similar to asking whether the stock market can be predicted. Qian and Rasheed (2004) showed that the Dow Jones Industrial Average (DJIA) could be more accurately predicted during periods when it obeyed certain statistical properties than at other times, and they used the Hurst exponent to partition the data. We can use the same exponent to look at R_Z and other solar and geomagnetic indices.

Hurst (1951) proposed what is now called the Hurst exponent (H) as a way to analyze the persistence or memory of a time series. His goal was to design the Aswan Dam by understanding the average flow rate of the Nile River as well as the extremes of the flow rates. A dam that was too big would spend most of its time standing empty while one that was too small would not prevent flooding of the downstream riverbed. It was also important to examine the behavior over many different timescales to discover any consistencies in the time dependence. The R_Z time series also has fluctuations with many timescales, and estimates of its predictability should include the influence of as many of those timescales as possible.

The Hurst exponent partitions a time series into three regions. i) $H = 0.5$ indicates a time series that behaves like a random walk (white noise). ii) $0 < H < 0.5$ indicates a time series that covers less distance than a random walk with a similar standard deviation. This means an increase is more likely to be followed by a decrease, and *vice versa*. The tendency to return to the mean increases as $H \to 0$. iii) $0.5 < H < 1$ indicates a time series that covers more distance than a random walk with a similar standard deviation, meaning that an increase tends to be followed by an increase and a decrease tends to be followed by a decrease. This tendency increases as $H \to 1$, which means that a series with a trend has $H \approx 1$.

Many economic, financial, and medical time series are persistent with $H > 0.5$. This has led to the development of several methods to estimate H. One popular method is the

rescaled range algorithm (or R/S analysis, see Mandelbrot and Wallis, 1969; Bassingth-waighte, Liebovitch, and West, 1994; Qian and Rasheed, 2004). For a time interval with N points, $X = X_1, X_2, \ldots, X_N$, the R/S analysis method requires you to calculate a set of quantities over a hierarchy of subintervals of the time series. These intervals will be described by the index t; $(R/S)_t$ is averaged over the subintervals $[X_1, X_m]$, $[X_{m+1}, X_{2m}]$ until $[X_{(t-1)m+1}, X_{tm}]$ where $m = \lfloor N/t \rfloor$ is the number of data points in each subinterval. Factors of two are convenient to partition the dataset, but the value of m must range from 10 to $N/2$ to sample all relevant timescales.

For each subinterval you calculate the following quantities:

i) The mean value of the j'th interval at resolution t ($\langle X \rangle_{t,j}$, m is the number of data points in each interval),

$$\langle X \rangle_{t,j} = \frac{1}{m} \sum_{i=jm+1}^{(j+1)m} X_i; \tag{1}$$

ii) The standard deviation series of each subinterval ($S_{t,j}$),

$$S_{t,j} = \sqrt{\frac{1}{m-1} \sum_{i=jm+1}^{(j+1)m} \left(X_i - \langle X \rangle_{t,j} \right)^2}, \quad j = 0, 2, \ldots, t-1; \tag{2}$$

iii) The mean adjusted series in each subinterval ($Y_{t,j}$),

$$Y_{t,j,i} = X_i - \langle X \rangle_{t,j}, \quad i = jm+1, \ldots, (j+1)m; \tag{3}$$

iv) The cumulative deviate in each subinterval ($Z_{t,j}$),

$$Z_{t,j} = \sum_{i=jm+1}^{(j+1)m} Y_{t,j,i}; \tag{4}$$

v) The range (maximum minus minimum) of $Z_{t,j}$ in each subinterval (R_t),

$$R_t = \max_j |Z_{t,j}| - \min_j |Z_{t,j}|; \tag{5}$$

vi) The rescaled range series (R/S) is formed by the ratios for all values of t,

$$(R/S)_t = R_t / S_t. \tag{6}$$

This process is summarized in Equation (7), where three levels of the hierarchy are shown for 16 data points. The bracketed points are used in the calculation of the respective $(R/S)_t$ by following steps $1-4$. These patterns can be extended to the right with more data points and vertically to encompass more levels of subintervals.

$$\underbrace{\underbrace{X_1, X_2, X_3, X_4,}_{(R/S)_{4,1}} \underbrace{X_5, X_6, X_7, X_8,}_{(R/S)_{4,2}}}_{(R/S)_{2,1}} \underbrace{\underbrace{X_9, X_{10}, X_{11}, X_{12},}_{(R/S)_{4,3}} \underbrace{X_{13}, X_{14}, X_{15}, X_{16}}_{(R/S)_{4,4}}}_{(R/S)_{2,2}} \tag{7}$$

$$\underbrace{\phantom{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}}}_{(R/S)_{1,1}}$$

The resemblance of the hierarchy in Equation (7) to wavelet analysis is shown in the use of wavelets to calculate H (Simonsen, Hansen, and Nes, 1998).

Hurst (1951) showed that $(R/S)_t$ scales as a power law in the timescale, or $(R/S)_t \sim ct^H$, where c is a constant. To estimate the Hurst exponent, we plot $\log(R/S)_t$ $versus$ $\log t$. The slope of the regression line approximates the Hurst exponent. Subintervals with more than

Figure 2 R/S analysis to obtain H for the sunspot number (R_z), $F_{10.7}$, and two geomagnetic indices Ap and aa. All daily values available for each index were used in the analysis. The number of points used in the analysis is listed in the third column of Table 1. A dashed vertical line is drawn at a time of 11 years (in days); the dotted lines are the log-log fits whose slopes are listed in the fourth column of Table 1. Dashed lines are drawn that correspond to $H = 0.5$ (lower, random variable) and $H = 1$ (upper, trending variable).
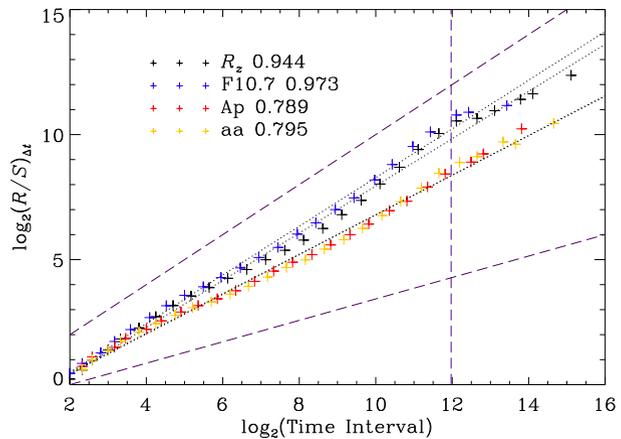
Table 1 Hurst exponents for solar and geomagnetic indices.

Index	Smoothing	No. of points	H	$D = 2 - H$	$\alpha = 2H - 1$
R_z	daily	69397	0.94	1.06	0.87
R_z	81-day	69397	0.96	1.04	0.92
R_z	365-day	69397	0.97	1.03	0.94
R_z	month	1908	0.91	1.09	0.82
R_z	annual	308	0.79	1.21	0.58
$F_{10.7}$	daily	21600	0.98	1.03	0.95
Ap	daily	28033	0.78	1.22	0.55
Ap	365-day	28033	0.99	1.01	0.97
aa	daily	51147	0.79	1.21	0.59

ten points were used to increase the accuracy of the subinterval averages. The example in Equation (7) produces a three-element series for $R/S = [(R/S)_4, (R/S)_2, (R/S)_1]$ that can be used in the regression line fit.

Figure 2 shows several examples of using an R/S analysis to derive the Hurst exponent of several well-known solar and geomagnetic indices. The slopes reported here average over the entire suite of locally derived values of H. A structure can be seen where the local slope varies due to the presence of periodic variations at that timescale (especially at 11 years, where the vertical line is drawn). This makes the calculation of H dependent on the length and sampling interval of the time series and, to a lesser extent, the method used to find the slope.

The Hurst exponents calculated by the R/S analysis for some solar and geomagnetic indices are listed in Table 1. The values do not agree with Kilcik *et al.* (2009), who quote 0.88 for the monthly sunspot number. After downloading the monthly file from the National Geophysical Data Center (NGDC) website,[1] the R/S analysis gave $H = 0.91$ for that file, above the 0.88 value of Kilcik *et al.* (2009), but in agreement at one significant digit.

Given the utility of H, it is not surprising that the Hurst exponent of R_Z was calculated as part of the predictions of Solar Cycle 24. For example, Suyal, Prasad, and Singh (2009)

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[1] ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/MONTHLY.

studied how H varied for R_Z and the radiocarbon sunspot proxy from Solanki *et al.* (2004). Chumak (2005) reports a smaller value of H for the Wolf sunspot number and describes the variation over the various subintervals.

The majority of Hurst exponents reported for R_Z are well above 0.5, indicating that some level of predictability exists in the data. But that could come from the presence of correlated changes in R_Z, especially the quasi-11-year period of the sunspot cycle (Suyal, Prasad, and Singh, 2009).

Several indices used to describe the variability of a time series can be derived from H. The fractal dimension (D) of a time series is related to H by $D = E + 1 - H$, where the Euclidean dimension of a 1D time series is $E = 1$. The spectral index in a Fourier power spectrum ($\propto 1/f^{\alpha}$) tends to have an exponent $\alpha = 2H - 1$ (Schepers, van Beek, and Bassingthwaighte, 1992). This means that a random walk series has $\alpha \approx 0$. When $H \to 0$ then $\alpha \approx -1$ (a power spectrum that is $\propto f$), indicating a series that returns to the mean, and $H \to 1$ gives $\alpha = 1$, the usual $1/f$ noise spectrum. (Geophysical time series do not always have $\alpha = 1$; see Gilman, Fuglister, and Mitchell, 1963.)

The Lyapunov exponent $\mathcal{L}_p$ that characterizes the solar dynamo cannot be derived from the 1D sunspot number time series. To derive $\mathcal{L}_p$ for a dynamical system it is necessary to estimate how many dimensions are required to describe the system and characterize the rate of separation of initially close trajectories. Greenkorn (2009) has described such calculations for a variety of solar activity indices. He concluded that $\mathcal{L}_p \sim 4$ for the solar dynamo but it varied from stochastic during Solar Cycles $10-19$ to chaotic for Solar Cycles $20-23$ and may now be returning to stochastic.

5. Prediction Categories

Early predictions of the amplitude and timing of Solar Cycle 24 are listed by Pesnell (2008) and Janssens (2006). These predictions were placed into categories of climatology, recent climatology (after Solar Cycle 17), precursor, dynamo model, spectral, and neural networks by Pesnell (2008) for further analysis.

5.1. Climatology

Climatological forecasts assume that the future of a system can be determined from the statistical properties of the past. Previous categorization schemes often labeled these statistical forecasts. An example is that R_{24} will be the average of all observed maxima. Using all of the numbered solar cycles in NOAA (2006), we calculate this as $R_{z,ave} = 115 \pm 40$. This also provides an error estimate for judging the predictions ($\sigma_0 = 40$). Timing information can be derived in a similar way.

The large number of forecasts in this category shows the utility of climatological forecasts. The largest and smallest predictions of R_{24} are in this class. The average of predictions in this category is very close to the actual climatological average. As described in Section 3, the climatological or statistical category of forecasts has tended to produce forecasts with a wide disparity in values.

Criticisms of climatological forecasts do exist. For example, Vaquero and Trigo (2008) describe how simple climatological analyses of solar cycle lengths are not predictive of the upcoming cycle. Without the additional information of the local variations in the solar dynamo, climatological forecasts appear to be investigations of the large deviations of previous solar cycles rather than useful forecasts.

5.2. Recent Climatology

Recent climatology refers to a forecast where future behavior is related to behavior in the recent past, defined here as including data since Solar Cycle 17 (about 1945). Examples include the "inertial" forecast that is used as a base forecast in weather forecasting, $R_{24} = R_{23}$, and the "even-odd cycle" forecast, $R_{24} = R_{22}$. Pesnell (2008) derived these forecasts for R_{24} using the information in NOAA (2006). Errors for these predictions were calculated as the standard deviation of the forecast and actual values (summed over the numbered solar cycles). If the sunspot number were a random variable, then the standard deviation of these predictions would be $\sqrt{2}\sigma_0$. As the standard deviations for both are smaller than this limit, the sunspot number maximum is not randomly distributed.

All four entries in this category predict above average activity in Cycle 24.

5.3. Precursor

Precursor forecasts, the leading indicators of solar activity, remain the most common category of predictions. Two types of precursors dominate this category:

i) Solar polar magnetic field at minimum $\approx$ level of activity at next maximum. The three predictions in this category tend to be near or below average for Cycle 24. Updates to these forecasts tended to give similar numbers; *i.e.*, the method converged. The Solar Dynamo Amplitude (SODA) index is a member of this class but has the unique ability to be continuously updated (Schatten, 2005).

ii) Geomagnetic activity near minimum is an indicator of the level of activity at next maximum. Nine of the 16 geomagnetic precursor predictions in Table 2 used aa as their indicator of geomagnetic activity, six used Ap, and one used both. As described by Pesnell (2011) and Pesnell (2009), these predictions are sensitive to the timing of the value selected as the precursor, and although the earliest predictions were for above-average levels of activity in Solar Cycle 24, many have been reduced after the minimum passed.

The remaining precursor predictions used solar properties such as global magnetic field and have a wide divergence in their forecasts.

5.4. Dynamo Model

Physics-based models capable of integrating conservation equations produce dynamo model forecasts; they can include data assimilation models. For the first time, predictions in this category are available. The two most complete models, by Dikpati, de Toma, and Gilman (2006) and Choudhuri, Chatterjee, and Jiang (2007), predict high and low solar activity, respectively. Cameron and Schüssler (2007) discuss the progress and problems in using these models for predictions of solar activity.

5.5. Spectral

A spectral forecast examines a Fourier analysis of the sunspot time series for invariant quantities such as frequencies whose amplitudes are conserved or have a simple time dependence. Wavelet-based and autoregressive forecasts are classified as spectral. Three autoregressive forecasts of R_{24} agree in predicting below-average activity for Solar Cycle 24.

Predictions based on the motion of the solar center of mass (represented by Charvátová, 2009) are classified as spectral because they look at periods of the planets as the main driver of the solar cycle.

Table 2 Predictions of Solar Cycle 24.

Predicted maximum		Category[a] and summary		Author and date	
R_{24}	Timing				
185	2010–2011	C	Projection of last five cycles (JSC)	Horstman	2005
180 ± 32	–	P	Disturbed days	Panel[b]	2007
180	2014	S	Modified global minimum analysis	Tsirulnik, Kuznetsova, and Oraevsky	1997
152–197	–	P	Integral of sunspot number	Podladchikova, Lefebvre, and Van der Linden	2006
155–180	–	D	Modified flux-transport dynamo model calibrated with historical run of sunspot area	Dikpati, de Toma, and Gilman	2006
160 ± 25	–	P	Analysis of aa index	Hathaway and Wilson	2006
160 ± 54	2010.6	R	$R_{24} = R_{22}$ (even-odd)	Pesnell	2008
150 ± 22	2011.04	R	Effective duration of Cycle 19	Du et al.[i]	2008
148	–	P	aa at minimum	Panel[b]	2007
145	12/2009	N	Neural network forecast	Maris and Oncica	2006
145 ± 30	2010	D	Fast meridional circulation speed during Cycle 22	Hathaway and Wilson	2004
145 ± 7	2011.2	S	Chaos theory (spectral)	Crosson and Binder[n]	2009
145	2011–2012	N	Spectral and neurofuzzy analysis	Gholipour et al.	2005
144	–	P	aa during decline of Cycle 23	Jain	2006
142 ± 24	–	P	aa at solar minimum	Kane	2007a
140.4 ± 15.7	5/2011	R	Running correlation coefficient between amplitude and cycle length	Du, Wang, and Zhang[j]	2008
140 ± 20	10/2011	–	Panel consensus prediction (high)	Panel[c]	2007
140	2012.5	P	Disturbed days analysis	Chopra and Dabas	2006
135 ± 20	–	P	aa/R_z precursor	Panel[b]	2007
134 ± 50	2011.7	C	Average of the last 8 solar cycles	Kennewell and Patterson	2006
133	2009.5	C	Statistics of $\sqrt{R_z}$	Tritakis, Mavromichalaki, and Giouvanellis	2006
130 ± 15	–	P	Complexity of Hα synoptic charts	Tlatov	2006
124 ± 30	–	P	Value of aa at solar minimum	Nevanlinna	2007
124 ± 23	–	P	Number of disturbed days in Ap	Dabas et al.	2008
124	–	C	Statistics of Equal Phase Average	Khramova, Krasotkin, and Kononovich	2002
122 ± 6	11/2010	C	Statistical analysis of cycle parameters	Kim, Wilson, and Cucinotta	2006
120 ± 60	2011.167	C	Modified McNish–Lincoln model (MSAFE)	Euler and Smith	2006
120 ± 45	2010.0	R	$R_{24} = R_{23}$ (inertial)	Pesnell	2008
120 ± 25	–	P	Behavior of aa	Panel[b]	2007
120 ± 20	2010	S	Periodicity of solar inertial motion	Charvátová[j]	2009
116 ± 13.2	2012–2013	S	Spectral analysis of R_z	Echer et al.	2004
115 ± 40	2011.3	C	$R_{24} = R_{z,ave}$ (average)	Pesnell	2008
115 ± 30	–	P	Number of disturbed days	Rabin	2007
115 ± 28	2010.5	P	Precursor + nonlinear dynamics	Sello[d]	2003
115 ± 15	–	P	Area of high-latitude unipolar regions	Tlatov	2006
115 ± 13	–	P	Large-scale magnetic field	Tlatov	2006
115 ± 6	–	P	Geomagnetic precursor (aa)	Ahluwalia[h]	2008
114.8 ± 17.4	–	C	Cycle $n + 1 \propto$ decline of $n - 2$	Du and Du	2006
114 ± 43	–	C	Mean of Cycles 1–23	Prochasta	2006

Table 2 (*Continued.*)

Predicted maximum		Category[a] and summary		Author and date	
R_{24}	Timing				
112	–	S	Combined empirical mode decomposition and autoregression	Xu *et al.*	2008
111 ± 18	–	P	Minimum value of Ap	Thompson	2008
110 ± 65	2/2011	C	Modified McNish–Lincoln model (MSAFE)	Euler and Smith	2006
110 ± 15	–	S	Transfer function model	de Meyer	2003
110 ± 11	2012	S	Autoregressive model	Hiremath	2008
110 ± 10	–	P	Dipole–octupole magnetic moments	Tlatov	2006
110	2011	N	Hybrid, neutral network and fuzzy logic	Quassim, Attia, and Elminir[j]	2007
109 ± 17	–	C	Rise time of cycle *vs.* R_{max}	Kane[i]	2008
108 ± 38	2011	C	Skewness of previous cycles separated into even/odd cycles	Lantos	2006
105 ± 10	–	P	Average of four precursor predictions	Obridko[l]	2008
105 ± 9	2010 – 2011	S	Extrapolation of dominant spectral components found by MEM	Kane	1999
101 ± 20	2012.5	S	Autoregressive, linear prediction	Pesnell	2008
$83.2 - 119.4$	3/2012	C	Statistics of solar cycles	Wang *et al.*[e]	2002
100.2 ± 7.5	2012.7	C	Similarity of SC 23 with previous cycles; correlation between risetime and maximum amplitude	Wang *et al.*[n]	2009
97 ± 25	–	P	Geomagnetic precursor (aa/solar wind speed)	Wang and Sheeley[n]	2009
96 ± 13	–	P	Geomagnetic precursor (Ap)	Kryachko and Nusinov[h]	2008
93 ± 21	2012	S	Theory of nonlinear dynamics	Baranovski, Clette, and Nollau[k]	2008
92 ± 24	–	S	Extrapolation of dominant spectral components found by MEM	Kane[i]	2007b
91.9 ± 27.9	1/2011	S	Auotregressive, moving average	Roth	2006
90.7 ± 9.2	–	P	Number of spotless days at minimum	Hamid and Galal	2006
90 ± 12	8/2012	–	Combined predictions	Brajša *et al.*[o]	2009
90 ± 10	8/2012	–	Panel consensus prediction (low)	Panel[c]	2007
87.8 ± 20.2	6/2013	C	Correlation of cycle length with succeeding maxima	Watari[l]	2008
87.5 ± 23.5	–	S	Wavelet analysis of sunspot maxima and aa minima modulations	Duhau	2003
87.4	2012.96	S	Sugihara–May chaos algorithm	Kilcik *et al.*[m]	2009
87 ± 7	–	P	Statistics of low-latitude sunspot groups and the north–south asymmetry in active region area	Javaraiah[k]	2008
85	–	D	Simplified dynamo model and Kalman filter	Kitiashvili and Kosovichev[i]	2008
80 ± 21	2012	S	Mathematical theory of nonlinear dynamics. Predicts a long cycle lasting 12 years	Baranovski, Clette, and Nollau	2008
80 ± 30	2012	P	Solar polar field precursor	Schatten	2005
80	–	D	Flux-transport dynamo model	Choudhuri, Chatterjee, and Jiang	2007

Table 2 (*Continued.*)

Predicted maximum		Category[a] and summary		Author and date	
R_{24}	Timing				
77 ± 15	–	C	Correlation of maximum with 4 quantities	Osherovich and Fainberg[l]	2008
76 ± 15	2012.8	S	Nonlinear model, result is average of two in paper	Aguirre, Letellier, and Maquet[h]	2008
74 ± 10	–	P	Statistics of low-latitude sunspot groups	Javaraiah	2007
70 ± 2	–	P	Polar magnetic field strength at solar minima	Svalgaard, Cliver, and Kamide[f]	2005
70 ± 17.5	12/2012	S	Statistical Gaussian-based extrapolation	Kontor	2006
65 ± 20	2015	P	Geomagnetic precursor combining Ap, F10.7, and a recurrence index	Pesnell[n]	2009
~ 60	7/2012	C	Statistics of radiocarbon isotopes	Miyahara[l]	2008
< 50	2010–2011	C	Statistics of the $\lambda 5303$ Å coronal line	Badalyan, Obridko, and Sykora	2001
42 ± 35	–	S	Periods in R_z and radiocarbon isotopic abundances	Clilverd *et al.*	2006
low	–	C	Observations of flare energy release during the descending phase of Cycle 23 (empirical)	Mariş *et al.*[g]	2004

[a]The third column is a one-letter description of the method, C – Climatology, D – Dynamo model, N – Neural network, P – Precursor, R – Recent climatology, or S – Spectral.

[b]These predictions were created during the panel deliberations (Biesecker and the Solar Cycle 24 Prediction Panel, 2007).

[c]These consensus predictions were created during the panel deliberations (Biesecker and the Solar Cycle 24 Prediction Panel, 2007) and were not placed into categories.

[d]This prediction is based on the method of Sello (2003) and was received 4 October 2006.

[e]The average of the predictions given by Wang *et al.* (2002).

[f]The prediction of Svalgaard, Cliver, and Kamide (2005) was updated at the panel meeting from 75 ± 8 to 70 ± 2.

[g]The predicted maximum of Mariş, Popescu, and Beşliu (2004) was set to 40 in Figure 3.

[h]Added after publication of original table: Ahluwalia (2008), Kryachko and Nusinov (2008), and Aguirre, Letellier, and Maquet (2008). The latter reported two predictions, 65 ± 16 and 87 ± 13, using a nonlinear autoregression technique.

[i]15 October 2008: Added Du *et al.* (2008), Kitiashvili and Kosovichev (2008), Kane (2007b) (update of Kane, 1999), Kane (2008).

[j]18 October 2008: Added Charvátová (2009) as the average of 140 based on international sunspot number and 100 based on group sunspot number; Quassim, Attia, and Elminir (2007), Du, Wang, and Zhang (2008).

[k]6 November 2008, Added two published predictions that update previously listed values: Javaraiah (2008), Baranovski, Clette, and Nollau (2008).

[l]8 December 2008: Added Miyahara (2008), Obridko and Shelting (2008), Osherovich and Fainberg (2008), and Watari (2008).

[m]2 January 2009: Added Kilcik *et al.* (2009).

[n]15 March 2009: Added Crosson and Binder (2009), Wang *et al.* (2009), Wang and Sheeley (2009), Pesnell (2009).

[o]6 April 2009: Added Brajša *et al.* (2009) but did not categorize. Combination of climatological, spectral, and ARMA. Prediction is for a low amplitude for Solar Cycle 24.

Some of the spectral models are based on a nonlinear time series analysis that includes an explicit dynamo model equation (see *inter alia*, Aguirre, Letellier, and Maquet, 2008; Ashrafi and Roszman, 1992; Volobuev and Makarenko, 2008). These models are categorized as "Spectral" in the current table, because much of the analysis relies on how Fourier transforms are treated rather than the actual dynamo model representation. Because the "Dynamo-model" category was designed to include models that integrated the conservation equations in space and time while the nonlinear models are averages of the conservation equations, this category definition should be reconsidered.

Forecasts in the spectral category tended to predict that Solar Cycle 24 will have slightly below-average activity. Only one was for a very high amplitude, while another provided the lowest quantitative prediction, the possibility that we will see the lowest solar activity since the Dalton Minimum in the early 1800s (Clilverd *et al.*, 2006).

5.6. Neural Network

Neural network forecasts are derived from nonlinear, statistical algorithms that determine and model complex relationships between inputs and outputs to find patterns in the data that can be extrapolated. Neural networks can be combined with other techniques, including spectral methods, to increase their accuracy. The two neural network forecasts for Solar Cycle 24 agree in their prediction of an above-average Cycle 24.

5.7. Stock Market and Economic Indicator Prediction Methods

Solar cycle prediction algorithms are similar to those used to predict the stock market and economic indexes. Economic and stock market prediction methods can be divided into several broad categories, which have some overlap with the solar cycle prediction categories. They are fundamental analysis, economic indicators, technical analysis, and technological methods.

Fundamental analysis: Fundamental analysis is concerned with the company that underlies the stock itself (Graham, 2003). They evaluate a company's past performance as well as the credibility of its accounts. Performance ratios, such as the P/E ratio, are used to assess the validity of a stock. Warren Buffett is perhaps the most famous fundamental analyst.

Economic indicators: A statistic that describes part of the economy. Leading indicators tend to change before the economy as a whole, lagging indicators change after, and coincident indicators change with the economy as a whole. Leading indicators are the precursors of economic forecasting. Part of the art is how the data is developed, and straight predictions are rare. For example, seasonal adjustments are necessary in the unemployment numbers to allow month-to-month comparisons. The cost-of-living index (CPI) is released every month with a flurry of explanations. That tends to diminish the usefulness of economic index algorithms in solar cycle predictions. The cosmic ray intensity is an example of a lagging indicator of solar activity.

Technical analysis: This determines the future price of a stock based only on the analysis of the trends of the past price (Lo and Mackinlay, 2002). Time series analysis techniques, such as Fourier and Hurst exponent analysis, are a major part of the technical analyst's toolbox. Patterns such as the head and shoulders or cup and saucer are used to describe and predict the data. Much emphasis is placed on how to smooth the economic data, such as the exponential moving average (EMA), which applies an ever-decreasing weight to points that are further back in time. One version, called the Holt–Winters forecasting procedure, is used to anticipate trends and seasonal variations in a time series (Chatfield and Yar, 1988;

Chatfield, 1978). Such methods could be adapted to analyzing the solar cycle variations in the presence of a trend in the envelope of solar activity.

Technological methods: Stock market predictions have become heavy users of computer algorithms. Neural networks are one common technique used to search for a relationship among indicators (Herbrich *et al.*, 1999). Another example is the Black–Scholes model for pricing derivatives. As first described by Black and Scholes (1973), this equation attempts to provide future price information for a risky investment in the presence of lower, but more dependable, returns from another investment.

Although there is no one-to-one relationship between the categories used in solar cycle predictions and those used to predict the stock market, some concepts may be transferable. What sets the solar cycle apart is an underlying system that can be represented by a physics-based model.

6. Predictions of Cycle 24

The 75 predictions in Table 2 are a combination of the predictions in Pesnell (2008) and others that appeared later. The table is organized by the predicted sunspot maximum (R_{24}) and includes R_{24}, 1-σ error, timing, category of prediction, a short summary of the method, and the reference. If a prediction did not have a category, one was assigned by the author. Predictions found or submitted since the publication of Pesnell (2008) are called out in the notes to Table 2. Other summary lists of predictions are given in Janssens (2006, 2005). Petrovay (2010) discusses some of the details in several of the prediction categories. The predicted maxima, uncertainties, and references are shown in Figure 3. Categories for each prediction are shown by a color coding listed in the legend.

The more recent predictions have reduced the average of all amplitude predictions to slightly below average while retaining the large standard deviation. Compared to Pesnell (2008), the average prediction in the geomagnetic precursor categories (aa and Ap) has decreased while the maximum prediction has increased in the spectral category.

Several predictions are listed by Obridko and Shelting (2008), who use an index of global field strength, a polar field precursor, and recurrence of geomagnetic activity (shifted back six years) to give three different predictions of the amplitude of Solar Cycle 24. The average is 108 ± 26 and a range of $80 - 131$. Obridko (2008) submitted another prediction using aa as a precursor.

Kilcik *et al.* (2009) continue the long tradition of using chaos theory to analyze the sunspot time series. Their prediction is for a low cycle ($R_{24} = 87.4$) with support from a correlation between rise time of a cycle and its amplitude. Additional support is provided by an estimate of the Hurst exponent (H) by a rescaled range analysis (see Section 3 above). Their estimate of $H \approx 0.88$ indicates that R_Z has sufficient persistence to allow a prediction to be made.

Bushby and Tobias (2007) claim that simplified dynamo models inferred from time series analyses that are based on chaos theory may be more accurate for solar cycle prediction then dynamo models. But the wide range of global parameters derived from such studies (summarized in Greenkorn, 2009) shows that this claim may also be optimistic. Estimates of the fractal dimension are as high as five, depending on the time interval considered.

During the minimum between Solar Cycles 23 and 24, the number of days when sunspots were not observed (number of spotless days) was a much-discussed parameter. Hamid and Galal (2006) proposed using the number of spotless days as a precursor of the amplitude of the upcoming sunspot maximum. Because they used data from well before solar minimum,
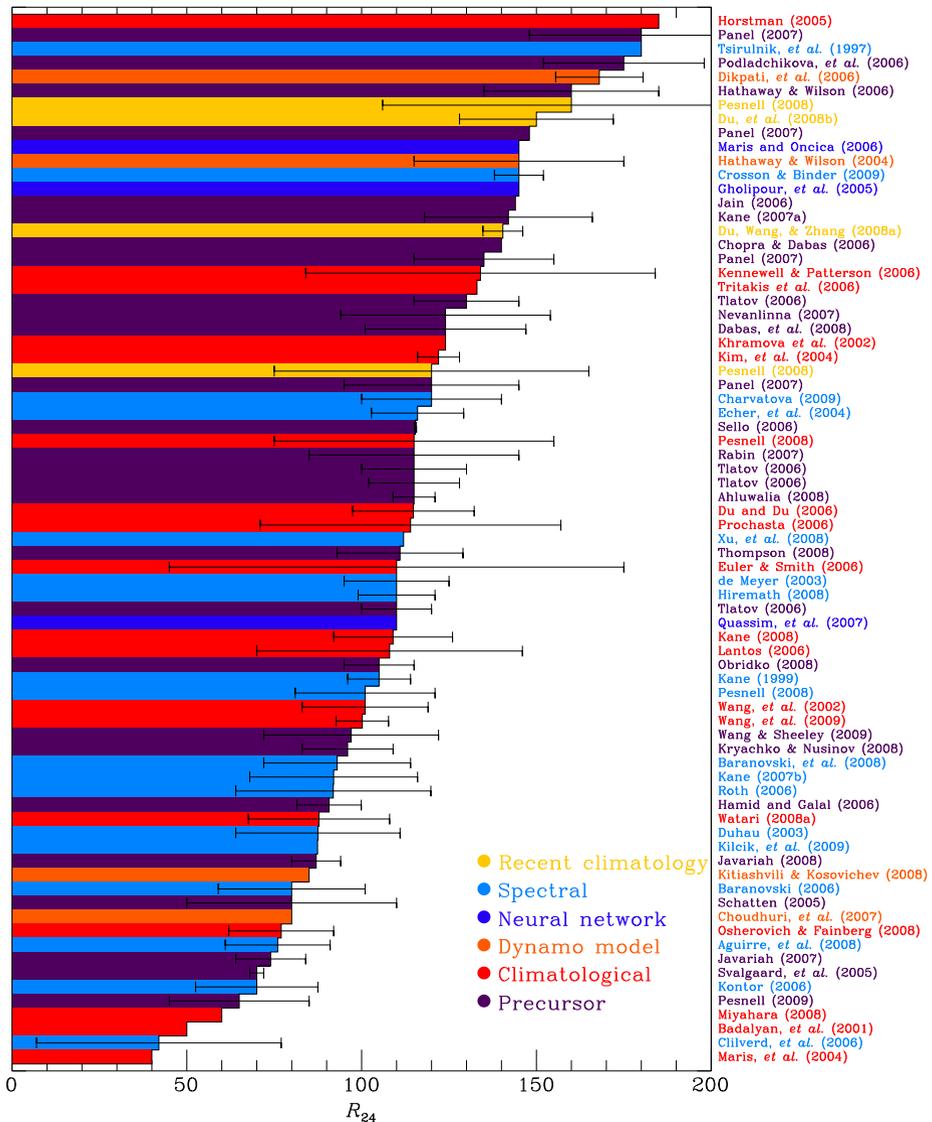
Figure 3 The predictions from Table 2, plotted in order of decreasing predicted maximum for Cycle 24. The prediction categories are color-coded as in the legend. Compared with Pesnell (2008) the distribution now appears to have an excess of low-amplitude predictions.

and the minimum was longer and had fewer sunspots than expected, this is explored in the Appendix. To be useful as a predictor, a stable estimate of the precursor must exist before solar minimum. Because the number of spotless days continued to increase after the minimum passed in December 2008, an algorithm is necessary to predict the number of spotless days before using that parameter to predict the amplitude of the upcoming cycle. Also, the number of spotless days appears to be biased to higher values before 1920. If only
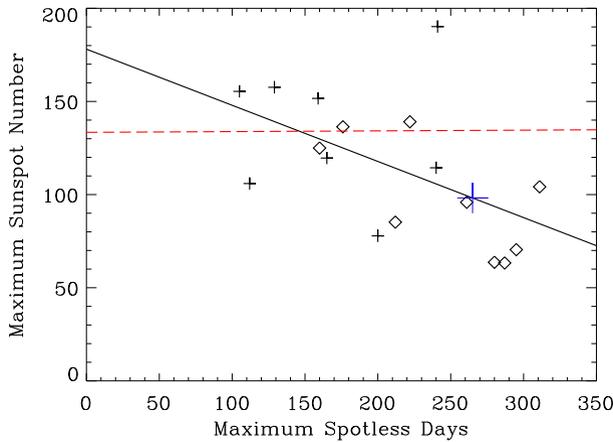
Figure 4 The maximum annual number of spotless days in each inter-cycle interval *vs.* maximum annual sunspot number of the following solar cycle. Values after 1830 but before 1920 are shown as diamonds and those after 1920 as plus signs. The solid line is a linear fit to the data and the blue cross is the prediction that $R_{24} = 100$ based on using a maximum annual spotless day count of 265 in 2008 in that fit. The red dashed line shows the linear fit if only data after 1920 is used, showing no correlation for the modern data. Created by solar/sunspots/plot_spotless_days.pro.

Table 3 Summary of predictions for Solar Cycle 24.

Category	Number	Average	Range
Average	23	115 ± 40	$49 - 202$
All	75	113 ± 32	$40 - 185$
Climatology (C)	18	105 ± 34	$40 - 185$
Recent Climatology (R)	4	143 ± 17	$120 - 160$
Dynamo Models (D)	4	120 ± 44	$80 - 168$
Spectral (S)	18	101 ± 30	$42 - 180$
Neural Network (N)	3	133 ± 20	$110 - 145$
Precursor (P)	28	118 ± 29	$70 - 180$
Geomagnetic	16	127 ± 26	$75 - 180$
aa	9	132 ± 19	$97 - 160$
Ap	7	120 ± 33	$75 - 180$
Solar	12	108 ± 31	$70 - 175$
Polar fields	3	88 ± 24	$70 - 115$
Other solar	9	111 ± 29	$74 - 175$

The column labeled "Average" lists the average and standard deviation within each category. The data in this table is shown in Figure 5.

recent data is used, there is no correlation between the number of spotless days at solar minimum and the level of activity in the next solar maximum (Figure 4).

A summary of the predictions by category is listed in Table 3. The columns show the category of the prediction, the number of predictions in each category from Table 2, the average, and standard deviation of the predictions within the category, and the range of the predictions. The first entry in Table 3 is the average of all predictions in Table 2. The precursor category is expanded into subcategories to explore the consistency within the subcategories.

Figure 5 The categorized predictions in Table 3. The dot is the average prediction in each category, the color bar is drawn at the 1σ error limits, and the error bars show the range of each category. Except for the breakouts of the precursor class, the colors correspond to those in Figure 2. The number of predictions in each category is written under the symbols. A dashed horizontal line is drawn at $R_{24} = 115$.
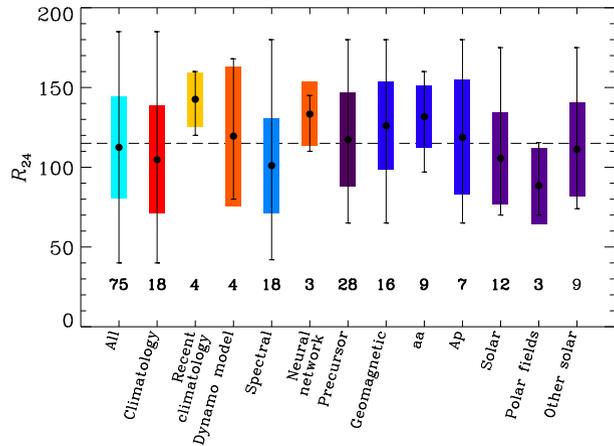
Figure 5 shows the categorized predictions; the standard deviation within the category is drawn as a colored box, and the range within the category is drawn as an error bar. The precursor category is also shown split into components to allow comparison of the various methods. A dashed line is drawn at $R_{24} = 115$, showing that almost all of the categories include $R_{z,\mathrm{ave}}$ in their predictions, with the aa precursor class the exception. The disagreement of the solar polar and geomagnetic precursors is large enough to recommend they be considered separate categories.

6.1. Timing Predictions

Few predictions are dedicated to the timing of the cycle. Again using the data in NOAA (2006), the average time between solar maxima is 11 ± 1.5 years, so Solar Cycle 24 should have peaked in April 2011, 11 years from the maximum of Solar Cycle 23. That this did not happen shows one of the pitfalls with statistical predictions – they include only averages of past behavior and no local information. In this case the minimum lasted longer than the 1σ error would indicate, and the upcoming maximum will occur 14 or more years after the previous maximum (*i.e.*, longer by 2σ). That there are odds of 1 in 22 of this occurring in a random variable illustrates the relatively few cycles for which there is good duration information.

This delay was explored by Dikpati (2008) using a flux-transport model driven by either sunspot data or polar field measurements. While the different input data predict large and small amplitudes of Solar Cycle 24, respectively, they both showed that the onset would be later than the statistical average.

Fyodorov, Klimenko, and Dovgalyuk (1996) describe a spectral method of predicting the timing of solar minimum. They predict that Solar Cycle 25 will start in early 2020 (2020.1). This would mean Solar Cycle 24 would last 12 years from their predicted start of 2008.2 (which was nine months early). Predictions of the timing of solar maximum have been less precise and usually depend on the timing of solar minimum. As we move into Cycle 24, those timing predictions that depend on the time of minimum and the shape of the rise will become more accurate.

6.2. Short-Term Predictions

Short-term predictions of the solar activity are possible once the new solar cycle has begun. One example is the MSAFE model based on the McNish–Lincoln algorithm (Niehaus, Euler, Jr., and Vaughan, 1996), used by Suggs, Euler, and Smith (2011) to predict that Solar Cycle 24 will have a double-peaked sunspot number. Their 50 % prediction has $F_{10.7}$ peaking at 130 ± 25 in December 2012 and R_{24} peaking at 77 ± 22 in August/September 2013. This lack of coincidence comes from the north–south asymmetry of the past few cycles, but the earlier peaking of $F_{10.7}$ in Solar Cycle 24 is the opposite of those cycles. Another short-term algorithm is the similar cycles method described by Solanki and Krivova (2011). These authors use Solar Cycle 14 as an exemplar for the new cycle to anticipate that $R_{24} = 60 - 100$. Thus, the short-term predictions show a convergence in their predicted amplitudes to a below-average cycle.

7. The Future: Solar Dynamo Models

In the 1950s the effort to forecast terrestrial weather and climate began in earnest (Lynch, 2008a). In the years since much effort has been made to improve the forecasts of temperature and precipitation for tomorrow and the next week. For example, Lynch (2008b) found that the range of accurate forecasts increases by about one day per decade. Weather models need to accurately track fronts and water in its various forms.

Similarly, solar dynamo models must accurately track the magnetic field of the Sun. Predictions using flux-transport dynamo models first appeared for Solar Cycle 24. In flux-transport models the motions of the plasma are imposed and the magnetic field response is calculated. Although the Solar Cycle 24 predictions of these models vary quite a bit, some information about how the Sun responds to the meridional transport speed and the magnetic diffusivity has been forthcoming (Nandy, Munoz-Jaramillo, and Martens, 2011). These authors found that changes in speed and phasing of meridional transport speed can explain cycle-to-cycle differences. How the predicted amplitude changes with different diffusivities and more realistic meridional flow profile must be understood.

As dynamo theories begin to explain fluctuations of the solar cycle, they will have to also address how the Sun gets into and out of grand minima. Should we be counting the number of cycles seen in the helioseismic record to signal the beginning of a grand minimum (Howe *et al.*, 2009)? Is there a clue in how the polar magnetic field cancels at maximum? The northern hemisphere is already showing that maximum is close, while the southern hemisphere remains in the rising phase of the cycle.

Relating the internal dynamo to the external observations remains an issue. What is the metric for comparing the theoretical calculations of the magnetic field with how we observe that field at and above the solar surface? Self-consistent nonlinear models of the solar cycle are discussed in Charbonneau (2010).

8. Beyond Solar Cycle 24

Some work has been done to estimate the level of solar activity over periods of time longer than five to ten years. One strategy is to predict the envelope of activity rather than the explicit sunspot number.

For example, Solanki *et al.* (2004) conclude that the Sun is in a era of rare high solar activity; this current high level has already lasted almost 65 years. They estimate that, given

the current high level and duration of that level of solar activity, there is a probability of $8\,\%^{+3\,\%}_{-4\,\%}$ that solar activity will remain above a sunspot number of 50 over the next 50 years. The probability that it will continue until the end of the twenty-first century is below 1 %. Solanki and Krivova (2011) also discuss the timing of grand minima and maxima within this dataset to show that we are just as likely to enter another grand maximum of solar activity in the next 30 years as a grand minimum. Ogurtsov (2005) analyzes the same ^{14}C record to show that solar activity in the next 50 years will be lower than in the twentieth century. He uses a nonlinear forecasting technique described in Ashrafi and Roszman (1992). Autoregressive techniques applied to the radiocarbon data (after detrending with a polynomial) give a decreased amplitude as well.

Clilverd *et al.* (2006) use a frequency modulation model of the 11-year Schwabe cycle in the sunspot record to estimate that the next two cycles will have below-average activity with a return to average activity for the remainder of the twenty-first century. Hathaway and Wilson (2006) predict a low level of activity in Solar Cycle 25. Javaraiah (2008) uses the evolution of the north–south asymmetry in active region timing to predict that Solar Cycle 25 will be more active than Solar Cycle 24.

One issue with longer term predictions arises from the finite length of the R_Z time series. Any spectral method (which includes autoregressive methods) that uses only the R_Z time series will tend to reproduce the known variations, especially in the statistical appearance of grand minima.

9. Looking to Solar Cycle 25

As fascinating as the predictions for Solar Cycle 24 have been, Solar Cycle 25 may be even more interesting. The solar polar fields at solar minimum have steadily decreased for the last three and possibly four minima. This means that the solar polar field precursors such as the SODA index (Schatten, 2005; Schatten and Pesnell, 1993) have never predicted an increased solar cycle amplitude. If the solar polar field continues to be a faithful precursor, then soon after the peak of Solar Cycle 24 the rebuilding of the polar field will indicate whether to anticipate a larger or smaller amplitude for Solar Cycle 25.

Solar activity predictions will continue in research and operational settings for the foreseeable future. There are indications that the need for solar predictions has moved from the science community to a global space weather user support system. For example, the NOAA Space Weather Prediction Center (SWPC), based in Boulder, Colorado, USA, supported the Solar Cycle 24 Consensus Prediction Panel and publishes updates on their website.[2] These predictions are made available at the ESA Space Weather Web Server.[3] Long-term predictions are described at the Chinese Solar Activity Prediction Center (He *et al.*, 2008), which uses the work described in Du and Du (2006), Du *et al.* (2008), Du, Wang, and Zhang (2008) for their predictions. Results from this center are also used by the Australian Space Weather Center.[4] The Russians have begun to standardize their prediction method to a geomagnetic precursor (Kryachko and Nusinov, 2008). Shorter term (up to several month) predictions are also available from the South African National Space Agency[5] and the South Korean gov-

[2]http://www.swpc.noaa.gov/SolarCycle/.

[3]http://www.esa-spaceweather.net/spweather/current_sw/index.html.

[4]http://www.ips.gov.au/Space_Weather.

[5]http://www.spaceweather.co.za.

ernment.[6] Except for SWPC, many of these agencies began operations during Solar Cycle 23 and reflect the need for any country with assets in space to monitor and predict solar activity and space weather to protect their satellites and technology.

After Solar Cycle 24 peaks and activity begins to subside, a new set of predictions for Solar Cycle 25 will appear. At some point one or more groups will convene to develop a consensus prediction for Solar Cycle 25. Here is a list of topics and questions for the creators of those predictions to consider:

i) New *a priori* estimates of the level of activity and standard deviation of the upcoming cycle; this should be announced along with the call for predictions. Pesnell (2008) used the climatological mean $R_{z,\text{ave}} = 115$ and $\sigma_0 = 40$ as *a priori* estimates.

ii) The call for predictions should include a request for the information necessary to rank a prediction using skill scores (Wilks, 1995) or a similar technique. The skill score compares the mean squared error of the forecast (MSE_{fc}) with the same from a reference forecast (MSE_{ref}) to give the forecast skill ($\text{SS} = 1 - \text{MSE}_{\text{fc}}/\text{MSE}_{\text{ref}}$). The skill score is closer to 1 for better forecasts. Pesnell (2008) used the significance of the prediction from the climatological mean as a way to rank the forecasts, but this has not been used to rate the predictions after the maximum has been reached.

iii) A standard set of solar and geomagnetic activity indices. Every prediction of solar or geomagnetic activity should be produced using the same set of data. Even if the method is developed for another calibration of the data, it should also be applied to the standard set. The time span of the dataset could be restricted, although this affects the ability to produce updatable forecasts. As an example, Charvátová (2009) reports predictions for R_{24} using R_Z or the group sunspot number that differ by σ_0.

iv) Provide standard estimates for the timing and amplitude of previous solar minimum and maximum, and a more physical definition of solar minimum. Many (if not most) of the predictions of the timing of solar maximum were based on the timing of solar minimum. Predictions that assumed solar minimum would occur in 2006 were invalidated when the minimum was late in arriving.

v) Can we devise a set of categories that more accurately partitions the predictions? The analysis presented here shows that the categories are either too broad in scope or that the predictions underestimate their uncertainty. There is a limit to the number of categories – each method being unique – but some guidance may be useful. The ultimate categories would contain consistent sets of predictions.

vi) Should the chaos-theory-guided results be a separate category?

Along with these topics, a few questions could become part of the charter of a Solar Cycle 25 Consensus Prediction Panel.

i) Should a category of magnetic field predictions be implemented? This enlarges the charter from predicting solar activity at Earth to a global heliospheric magnetic field.

ii) Should criticisms of prediction methods be a part of the panel's portfolio? Espousing criticisms can limit the methods that are submitted to the panel as well as create a time sink for the panel discussions. The working group dedicated to long-term predictions of Solar Cycle 22 did publish reports that include some critical discussion (McIntosh *et al.*, 1979; Brown, 1986). However, their conclusion that statistical techniques produce a large spread in their predictions is still seen in such predictions for Solar Cycle 24.

iii) How should predictions with multiple categories from the same author be handled?

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[6]http://www.spaceweather.go.kr.

iv) How should updated predictions be handled; are the older predictions discarded in favor of the new ones? There is some cachet in producing the earliest correct prediction, but is it more important to simply be correct? For example, Hathaway (2010) describes some deficiencies in the various geomagnetic precursor predictions that resulted in large changes in earlier predictions as Solar Cycle 23 ended.

10. Conclusions

Predictions of the sunspot number have been made since the cycle was discovered. The sunspot number and the geomagnetic activity indices have been used as examples for many types of time series analyses, including the development of the autoregression analysis and chaos theory. Analyses of the sunspot number series with the Hurst exponent and recent climatology predictions show that R_Z may be predictable on timescales of the solar cycle. However, simple time series analyses have not produced accurate predictions. In fact, the convergence of the climatology predictions to $R_{z,ave}$ probably reflects the large variation in the peak of the solar cycles and the differing sampling of the noise properties by the different algorithms. Also, the solar dynamo is a complex system whose variability is not fully represented by sunspot number alone. Coronal holes, high-speed streams, and other observed phenomena that do not track R_Z also arise from the dynamo. Each type of data can be used to help understand and predict solar activity.

Precursors were a major contributor to the consensus prediction of Solar Cycle 23 (Joselyn *et al.*, 1997). We have shown that the precursor category must be further classified as solar and geomagnetic to produce equivalent classes, illustrating the poor overlap of the two techniques. As a consequence of this divergence, the solar and geomagnetic precursors could be considered as separate categories. Part of the discrepancy could be related to the nature of the precursors. The SODA index is designed to be continuously updated; geomagnetic precursors must remove the remnant activity of the previous cycle and extrapolate the value of the precursor to minimum before using it in a prediction. Given the long decline of Solar Cycle 23 and the extremely low values eventually reached by the geomagnetic indices, the geomagnetic precursor predictions made several years before solar minimum were probably biased to high values.

It is possible that the future of solar activity predictions lies in the development of large-scale models of the solar dynamo that track the magnetic field within the Sun and as it is expelled from the Sun. Many models of the solar dynamo exist, from simple nonlinear characterizations to large numerical simulations, but none has shown a predictive capability. In fact, the large discrepancy in the Solar Cycle 24 predictions by flux-transport models shows that those models do not yet possess a predictive capability.

We have seen part of the solution: the combination of helioseismology and large-scale numerical models. Helioseismic imaging of active region emergence (Ilonidis, Zhao, and Kosovichev, 2011), subsurface flows (see, *inter alia*, Hindman *et al.*, 2004), and far-side imaging (González Hernández, Hill, and Lindsey, 2007) provide the data that both validate the large-scale models and provide input data to move toward assimilative methods.

Even if long-term predictions of solar activity become accurate and timely, forecasting exceptional events will always be difficult. The conditions that lead to solar radio bursts that overwhelm GPS (6 Dec. 2006), solar particle events that reach Earth without warning (ground level enhancements or GLEs), and triggers that span large distances on the Sun (also called nonlocal behavior, see Schrijver and Title, 2011) will challenge even the most daring forecaster. Improving such forecasting requires the development of near-real-time models that resolve the loops and other structures of the corona and chromosphere.

Figure 6 The number of spotless days in each calendar year from 1818 through mid-2011 for the NGDC International Sunspot Number dataset is shown as the black curve. Annual-averaged sunspot number is shown in red. Created by solar/sunspots/plot_spotless_days.pro.
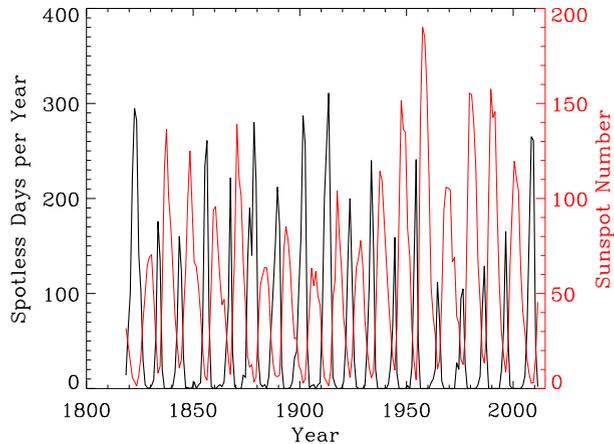
At this time the SODA index, which can be continually updated until solar minimum, continues to predict the same low amplitude that was calculated by Schatten (2005). This prediction is shown in Figure 1, where a peak of $R_{24} = 85$ is reached in June 2013, in agreement with the short-term predictions listed in Section 6.2.

As we become a society that travels to other planets, we will have to provide forecasts of the solar activity that would be seen at any point in the solar system. Given the wide range of the predictions for the amplitude of Solar Cycle 24, and the many methods that were used to produce them, we look forward to this cycle answering important questions about how to predict solar activity at the Earth and throughout the solar system.

Appendix: Spotless Days

The term "spotless days" refers to a count of the number of days in an interval during which no sunspots were observed. The number of spotless days reaches a maximum near solar minimum and is a candidate both for determining the timing of solar minimum and as a precursor of the amplitude of the upcoming cycle. The current minimum had more spotless days than any minimum since 1920.

Is the number of spotless days in the minimum between Solar Cycles 23 and 24 an exception? Figure 6 shows the number of spotless days in each year from the NGDC International Sunspot Number dataset.[7] This file contains data for 70795 days from 1 Jan. 1818 through 31 Oct. 2011. There are 10547 days (15 %) with a sunspot number of zero (the most recent on 14 Aug. 2011) and 3247 days (5 %) without a recorded value. The latter days are not included in the spotless day analysis. The red line in Figure 6 is the annual-averaged sunspot number.

Some notes on data completeness can be derived from this figure. In all maxima since 1920 there are years at solar maximum when no spotless days are recorded. Maxima between 1850 and 1920 do not share this behavior, with spotless days appearing in some Carrington

[7]ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/INTERNATIONAL/daily/RIDAILY.PLT.

rotations near solar maximum. Observations before 1850 are very incomplete, with significant fractions of a year not having visible spots even near solar maximum. This indicates that the observations present an incomplete picture of the count of spotless days until Solar Cycle 15.

Is there a correlation between the number of spotless days in a solar cycle and the number of sunspots in either of the surrounding maxima? Figure 4 shows the correlation plot between the maximum annual number of spotless days as the abscissa and the maximum annual sunspot number as the ordinate. A linear fit of $R_n = 178 - 0.30\,SP_n$ gives $R_{24} = 100$ for $SP_{24} = 265$. This prediction required data one year past solar minimum to guarantee that the maximum in spotless days had passed.

After culling the data to include cycles after 1920, we only have seven points of similar totals to examine. If only that data is used, there is little correlation between the annual number of spotless days at solar minimum and the level of activity in the next solar maximum (see the dashed line in Figure 4).

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